

# FORMULARIO DI TRIGONOMETRIA

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## Relazioni fondamentali:

$$1. \quad \sin^2(\alpha) + \cos^2(\alpha) = 1 \quad \forall \alpha \in \mathbb{R}$$

$$2. \quad \tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} \quad \forall \alpha \neq \frac{\pi}{2} + k\pi$$

$$3. \quad \cot(\alpha) = \frac{\cos(\alpha)}{\sin(\alpha)} \quad \forall \alpha \neq k\pi$$

$$4. \quad \cot(\alpha) = \frac{1}{\tan(\alpha)} \quad \forall \alpha \neq k\frac{\pi}{2}$$

## Archi opposti ed esplementari

(simmetrici rispetto all'asse x)

$$\sin(-\alpha) = -\sin(\alpha)$$

$$\cos(-\alpha) = \cos(\alpha)$$

$$\tan(-\alpha) = -\tan(\alpha)$$

$$\cot(-\alpha) = -\cot(\alpha)$$

## Archi che differiscono di $2\pi$

(congruenti)

$$\sin(2\pi + \alpha) = \sin(\alpha)$$

$$\cos(2\pi + \alpha) = \cos(\alpha)$$

$$\tan(2\pi + \alpha) = \tan(\alpha)$$

$$\cot(2\pi + \alpha) = \cot(\alpha)$$

## Archi supplementari

(simmetrici rispetto all'asse y)

$$\sin(\pi - \alpha) = \sin(\alpha)$$

$$\cos(\pi - \alpha) = -\cos(\alpha)$$

$$\tan(\pi - \alpha) = -\tan(\alpha)$$

$$\cot(\pi - \alpha) = -\cot(\alpha)$$

## Archi che differiscono di $\pi$

(simmetrici rispetto all'origine, ossia ruotati di  $180^\circ$ )

$$\sin(\pi + \alpha) = -\sin(\alpha)$$

$$\cos(\pi + \alpha) = -\cos(\alpha)$$

$$\tan(\pi + \alpha) = \tan(\alpha)$$

$$\cot(\pi + \alpha) = \cot(\alpha)$$

## Archi complementari

(simmetrici rispetto alla bisettrice del I e III quadrante)

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos(\alpha)$$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin(\alpha)$$

$$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot(\alpha)$$

$$\cot\left(\frac{\pi}{2} - \alpha\right) = \tan(\alpha)$$

## Archi che differiscono di $\pi/2$

(ruotati di  $90^\circ$ )

$$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos(\alpha)$$

$$\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin(\alpha)$$

$$\tan\left(\frac{\pi}{2} + \alpha\right) = -\cot(\alpha)$$

$$\cot\left(\frac{\pi}{2} + \alpha\right) = -\tan(\alpha)$$

## Archi la cui somma è $3\pi/2$

(simmetrici rispetto alla bisettrice del II e IV quadrante)

$$\sin\left(\frac{3}{2}\pi - \alpha\right) = -\cos(\alpha)$$

$$\cos\left(\frac{3}{2}\pi - \alpha\right) = -\sin(\alpha)$$

$$\tan\left(\frac{3}{2}\pi - \alpha\right) = \cot(\alpha)$$

$$\cot\left(\frac{3}{2}\pi - \alpha\right) = \tan(\alpha)$$

## Archi che differiscono di $3\pi/2$

(ruotati di  $-90^\circ$ )

$$\sin\left(\frac{3}{2}\pi + \alpha\right) = -\cos(\alpha)$$

$$\cos\left(\frac{3}{2}\pi + \alpha\right) = \sin(\alpha)$$

$$\tan\left(\frac{3}{2}\pi + \alpha\right) = -\cot(\alpha)$$

$$\cot\left(\frac{3}{2}\pi + \alpha\right) = -\tan(\alpha)$$

## FORMULARIO DI TRIGONOMETRIA

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### Formule di addizione

$$\sin(\alpha + \beta) = \sin(\alpha) \cdot \cos(\beta) + \cos(\alpha) \cdot \sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha) \cdot \cos(\beta) - \sin(\alpha) \cdot \sin(\beta)$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha) \cdot \tan(\beta)}$$

$$\cot(\alpha + \beta) = \frac{\cot(\alpha) \cdot \cot(\beta) - 1}{\cot(\alpha) + \cot(\beta)}$$

### Formule di sottrazione

$$\sin(\alpha - \beta) = \sin(\alpha) \cdot \cos(\beta) - \cos(\alpha) \cdot \sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha) \cdot \cos(\beta) + \sin(\alpha) \cdot \sin(\beta)$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha) \cdot \tan(\beta)}$$

$$\cot(\alpha - \beta) = \frac{\cot(\alpha) \cdot \cot(\beta) - 1}{\cot(\alpha) + \cot(\beta)}$$

### Formule di duplicazione

$$\sin(2\alpha) = 2 \cdot \sin(\alpha) \cdot \cos(\alpha)$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$$

$$\tan(2\alpha) = \frac{2 \cdot \tan(\alpha)}{1 - \tan^2(\alpha)}$$

$$\cot(2\alpha) = \frac{\cot^2(\alpha) - 1}{2 \cdot \cot(\alpha)}$$

### Formule di bisezione

$$\sin\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 - \cos(\alpha)}{2}}$$

$$\cos\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 + \cos(\alpha)}{2}}$$

$$\tan\left(\frac{\alpha}{2}\right) = \frac{\sin(\alpha)}{1 + \cos(\alpha)}$$

$$\cot\left(\frac{\alpha}{2}\right) = \frac{\sin(\alpha)}{1 - \cos(\alpha)}$$

### Formule di Werner

$$\sin(\alpha) \cdot \sin(\beta) = \frac{1}{2}(\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

$$\sin(\alpha) \cdot \cos(\beta) = \frac{1}{2}(\sin(\alpha - \beta) - \sin(\alpha + \beta))$$

$$\cos(\alpha) \cdot \cos(\beta) = \frac{1}{2}(\cos(\alpha - \beta) + \cos(\alpha + \beta))$$

### Formule di Prostaferesi

$$\sin(\alpha) + \sin(\beta) = 2 \cdot \sin(\lambda) \cdot \cos(\mu)$$

$$\cos(\alpha) + \cos(\beta) = 2 \cdot \cos(\lambda) \cdot \cos(\mu)$$

$$\sin(\alpha) - \sin(\beta) = 2 \cdot \cos(\lambda) \cdot \sin(\mu)$$

$$\cos(\alpha) - \cos(\beta) = 2 \cdot \sin(\lambda) \cdot \sin(\mu)$$

essendo  $\lambda = \frac{\alpha + \beta}{2}$  e  $\mu = \frac{\alpha - \beta}{2}$

$$\tan(\alpha) + \tan(\beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha) \cdot \cos(\beta)}$$

$$\tan(\alpha) - \tan(\beta) = \frac{\sin(\alpha - \beta)}{\cos(\alpha) \cdot \cos(\beta)}$$

$$\cot(\alpha) + \cot(\beta) = \frac{\sin(\alpha + \beta)}{\sin(\alpha) \cdot \sin(\beta)}$$

$$\cot(\alpha) - \cot(\beta) = \frac{-\sin(\alpha - \beta)}{\sin(\alpha) \cdot \sin(\beta)}$$

### Formule Parametriche

$$\sin(\alpha) = \frac{2t}{1+t^2} \quad \cos(\alpha) = \frac{1-t^2}{1+t^2} \quad \tan(\alpha) = \frac{2t}{1-t^2} \quad \cot(\alpha) = \frac{1-t^2}{2t}$$

essendo  $t = \tan\left(\frac{\alpha}{2}\right)$