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$$2\log_{\frac{2}{3}} x + \log_{\frac{2}{3}} 3 = \log_{\frac{2}{3}} (5x-2)$$

C.A. $\begin{cases} x>0 \\ 5x-2>0 \end{cases} \to \begin{cases} x>0 \\ x>\frac{2}{5} \end{cases} \to x>\frac{2}{5}$

$$\log_{\frac{2}{3}} x^2 + \log_{\frac{2}{3}} 3 = \log_{\frac{2}{3}} (5x-2)$$

$$\log_{\frac{2}{3}} 3x^2 = \log_{\frac{2}{3}} (5x-2)$$

$$3x^2 = 5x-2$$

$$3x^2-5x+2=0 \qquad x_{1,2} = \frac{5\pm1}{6} = \begin{cases} \frac{2}{3} & \text{accett.} \\ 1 & \text{accett.} \end{cases}$$

$S=\left\{\frac{2}{3}; 1\right\}$

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$$\log_3 (x^2+x) - \log_3 (x^2-x) = 1$$

C.A. $\begin{cases} x^2+x>0 \\ x^2-x>0 \end{cases} \to \begin{cases} x<-1 \ \vee \ x>0 \\ x<0 \ \vee \ x>1 \end{cases}$

$x<-1 \ \vee \ x>1$

$$\log_3 (x^2+x) = \log_3 3 + \log_3 (x^2-x)$$

$$\log_3 (x^2+x) = \log_3 (3x^2-3x)$$

$$x^2+x = 3x^2-3x$$

$$2x^2-4x=0$$

$$2x(x-2)=0$$

$x_1=0$ non accett.
$x_2=2$ accett.

$S=\{2\}$

$$\log_5 (x^2-4) - \log_5 (x+2) = 2$$

C.A. $\begin{cases} x^2-4>0 \\ x+2>0 \end{cases} \to \begin{cases} x<-2 \ \vee \ x>2 \\ x>-2 \end{cases}$

$x>2$

$$\log_5 (x^2-4) = \log_5 25 + \log_5 (x+2)$$

$$\log_5 (x^2-4) = \log_5 25(x+2)$$

$$x^2-4 = 25x+50$$

$$x^2-25x-54=0$$

$$(x-27)(x+2)=0$$

$x_1=27$ accett.
$x_2=-2$ NON accett.

$S=\{27\}$

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$$2\log x - \log(x-1) = 2\log 2$$

C.A. $\begin{cases} x>0 \\ x-1>0 \end{cases} \quad \begin{cases} x>0 \\ x>1 \end{cases} \to x>1$

$$\log x^2 = \log 4 + \log(x-1)$$

$$\log x^2 = \log 4(x-1)$$

$$x^2 = 4x-4$$

$$x^2-4x+4=0$$

$$(x-2)^2=0$$

$$x=2$$

$S=\{2\}$

$$2+\log_2 x = \log_2 7$$

$$\log_2 4 + \log_2 x = \log_2 7$$

C.A. $x>0$

$$\log_2 4x = \log_2 7$$

$$4x = 7$$

$x=\frac{7}{4}$ accett.

$S=\left\{\frac{7}{4}\right\}$

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$\log_6 (9x^2-1) - \log_6 (3x-1) = 1$

C.A. $\begin{cases} 9x^2-1>0 \\ 3x-1>0 \end{cases} \to \begin{cases} x<-\frac{1}{3} \vee x>\frac{1}{3} \\ x>\frac{1}{3} \end{cases}$

$x>\frac{1}{3}$

$\log_6 (9x^2-1) = 1 + \log_6 (3x-1)$

$\log_6 (9x^2-1) = \log_6 (18x-6)$

$9x^2-1 = 18x-6$

$9x^2-18x+5=0$

$x_{1,2} = \frac{9\pm6}{9} = \begin{cases} \frac{1}{3} & \text{non accett.} \\ \frac{5}{3} & \text{accett.} \end{cases}$

$S=\left\{\frac{5}{3}\right\}$

$2\log_2 x = 2 + \log_2 (x-1)$

C.A. $\begin{cases} x>0 \\ x-1>0 \end{cases} \to \begin{cases} x>0 \\ x>1 \end{cases} \to x>1$

$\log_2 x^2 = \log_2 4 + \log_2 (x-1)$

$\log_2 x^2 = \log_2 (4x-4)$

$x^2-4x+4=0$

$(x-2)^2=0$

$x=2$ accett.

$S=\{2\}$

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$\log (10-2x) = \log (5-x) - \log 4$

C.A. $\begin{cases} 10-2x>0 \\ 5-x>0 \end{cases} \to \begin{cases} x<5 \\ x<5 \end{cases} \to x<5$

$10-2x = \frac{5-x}{4}$

$40-8x = 5-x$

$-7x=-35 \to x=5$ non accett. $S=\emptyset$

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$\log_2 x - \log_2 (x+4) + 1 = 0$

C.A. $\begin{cases} x>0 \\ x+4>0 \end{cases} \to \begin{cases} x>0 \\ x>-4 \end{cases} \to x>0$

$\log_2 x + \log_2 2 = \log_2 (x+4)$

$2x = x+4$

$x=4$ accett. $S=\{4\}$

$\log (x-16) = \log 105 - \log x$

C.A. $\begin{cases} x-16>0 \\ x>0 \end{cases} \to \begin{cases} x>16 \\ x>0 \end{cases} \to x>16$

$\log (x-16) + \log x = \log 105$

$\log (x^2-16x) = \log 105$

$x^2-16x=105$

$x^2-16x-105=0$

$x_{1,2} = 8\pm13 = \begin{cases} -5 & \text{non accett.} \\ 21 & \text{accett.} \end{cases}$ $S=\{21\}$

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$2\log x - \log (2x+1) + \log 3 = \log (x-2)$

C.A. $\begin{cases} x>0 \\ 2x+1>0 \\ x-2>0 \end{cases} \quad \begin{cases} x>0 \\ x>-\frac{1}{2} \\ x>2 \end{cases} \to x>2$

$\log x^2 + \log 3 = \log (x-2) + \log (2x+1)$

$\log 3x^2 = \log (2x^2-3x-2)$

$3x^2 = 2x^2-3x-2$

$x^2+3x+2=0$

$x_{1,2} = \frac{-3\pm1}{2} = \begin{cases} -2 & \text{non accett.} \\ -1 & \text{non accett.} \end{cases}$ $S=\emptyset$

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$$\log_2(3x+1) - \log_2(x+2) + 2 = \log_2(9x-4) - \log_2 x$$

$$\log_2(3x+1) + \log_2 4 + \log_2 x = \log_2(x+2) + \log_2(9x-4)$$

$$\log_2(12x^2+4x) = \log_2(9x^2+14x-8)$$

$$12x^2+4x = 9x^2+14x-8$$

$$3x^2-10x+8=0$$

$$x_{1,2} = \frac{5\pm1}{3} = \begin{cases} \frac{4}{3} & \text{accett.} \\ 2 & \text{accett.} \end{cases} \qquad S=\left\{\frac{4}{3}\,;2\right\}$$

C.A.

$$\begin{cases} 3x+1>0 \\ x+2>0 \\ 9x-4>0 \\ x>0 \end{cases} \quad \begin{cases} x>-\frac{1}{3} \\ x>-2 \\ x>\frac{4}{9} \\ x>0 \end{cases} \rightarrow x>\frac{4}{9}$$

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$$\log(x^3-x^2-2x) = \log(x+\sqrt{2}) + \log(x-\sqrt{2}) + \log(x-2)$$

$$\log(x^3-x^2-2x) = \log(x^3-2x^2-2x+4)$$

$$\cancel{x^3}-x^2-2x = \cancel{x^3}-2x^2-2x+4$$

$$x^2=4$$

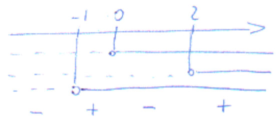
$$x = \begin{cases} -2 & \text{non accett.} \\ +2 & \text{non accett.} \end{cases}$$

$$S=\emptyset$$

C.A.

$$\begin{cases} x(x^2-x-2)>0 \\ x+\sqrt{2}>0 \\ x-\sqrt{2}>0 \\ x-2>0 \end{cases} \quad \begin{cases} -1<x<0 \vee x>2 \\ x>-\sqrt{2} \\ x>\sqrt{2} \\ x>2 \end{cases}$$

$$x(x-2)(x+1)>0$$



C.A. $\rightarrow$ $x>2$

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$$\log(4x-1) - \log(3x-1) = \log(1+x) - \log(1-x)$$

$$\log(4x-1) + \log(1-x) = \log(1+x) + \log(3x-1)$$

$$\log(4x-4x^2-1+x) = \log(3x-1+3x^2-x)$$

$$\log(-4x^2+5x-1) = \log(3x^2+2x-1)$$

$$-4x^2+5x-1 = 3x^2+2x-1$$

$$7x^2-3x=0 \qquad x_1=0 \quad \text{non accett.}$$

$$x(7x-3)=0 \qquad x_2=\frac{3}{7} \quad \text{accett.} \qquad S=\left\{\frac{3}{7}\right\}$$

C.A.

$$\begin{cases} 4x-1>0 \\ 3x-1>0 \\ 1+x>0 \\ 1-x>0 \end{cases} \rightarrow \begin{cases} x>\frac{1}{4} \\ x>\frac{1}{3} \\ x>-1 \\ x<1 \end{cases} \rightarrow \frac{1}{3}<x<1$$

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$$\frac{1}{3}\log(x^3-8x+5) = \log(x-1)$$

$$\log(x^3-8x+5) = 3\log(x-1)$$

$$\log(x^3-8x+5) = \log(x-1)^3$$

$$x^3-8x+5 = x^3-3x^2+3x-1$$

$$3x^2-11x+6=0$$

$$x_{1,2} = \frac{11\pm7}{6} = \begin{cases} \frac{2}{3} & \text{non accett.} \quad \frac{2}{3}-1<0 \\ 3 & \text{accett.} \rightarrow \frac{1}{3}\log 8 = \log 2 \end{cases}$$

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$\log 3 + \frac{1}{2}\log(x^2-2) = \log(6-x^2)$

$2\log 3 + \log(x^2-2) = 2\log(6-x^2)$

$\log 9 + \log(x^2-2) = \log(36-12x^2+x^4)$

$9x^2-18 = x^4-12x^2+36$

$x^4-21x^2+54=0$

$(x^2-3)(x^2-18)=0$

$x = -\sqrt{3}$ accett.; $+\sqrt{3}$ accett.; $-\sqrt{18}$ non accett.; $+\sqrt{18}$ non accett.

C. A. $\begin{cases} x^2-2>0 \\ 6-x^2>0 \end{cases}$ $\begin{cases} x<-\sqrt{2} \vee x>\sqrt{2} \\ -\sqrt{6}<x<\sqrt{6} \end{cases}$

$-\sqrt{6}<x<-\sqrt{2} \vee \sqrt{2}<x<\sqrt{6}$

$S=\{-\sqrt{3}; +\sqrt{3}\}$

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$\frac{1}{2}\log(x-\sqrt{5}) = \log(7-x^2) - \frac{1}{2}\log(x+\sqrt{5})$

$\frac{1}{2}\left(\log(x-\sqrt{5})+\log(x+\sqrt{5})\right) = \log(7-x^2)$

$\log(x^2-5) = \log(49-14x^2+x^4)$

$x^2-5 = 49-14x^2+x^4$

$x^4-15x^2+54=0$

$(x^2-9)(x^2-6)=0$

$x = 3$ non accett.; -3 non accett.; $\sqrt{6}$ accett.; $-\sqrt{6}$ non accett.

C.A. $\begin{cases} x-\sqrt{5}>0 \\ 7-x^2>0 \\ x+\sqrt{5}>0 \end{cases}$ $\begin{cases} x>\sqrt{5} \\ -\sqrt{7}<x<\sqrt{7} \\ x>-\sqrt{5} \end{cases}$

$\sqrt{5}<x<\sqrt{7}$

$S=\{\sqrt{6}\}$

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$\log 5 - \log(3+\sqrt{x}) = \log(3-\sqrt{x})$

$\log 5 = \log(3+\sqrt{x}) + \log(3-\sqrt{x})$

$\log 5 = \log(9-x)$

$5 = 9-x \rightarrow x=4$ accett. $S=\{4\}$

C.A. $\begin{cases} 3+\sqrt{x}>0 \\ 3-\sqrt{x}>0 \end{cases}$ $0\leq x<9$

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$\log(1-x) - \log(1+x) + \frac{1}{2}\{\log(1+2x) - \log(1-2x)\} = 0$

$2\log(1-x) - 2\log(1+x) + \log(1+2x) - \log(1-2x) = 0$

$\log(1-x)^2(1+2x) = \log(1+x)^2(1-2x)$

$(1-x)^2(1+2x) = (1+x)^2(1-2x)$

$(x^2-2x+1)(2x+1) = (x^2+2x+1)(1-2x)$

$2x^3+\cancel{x^2}-\cancel{4x^2}-\cancel{2x}+\cancel{2x}+\cancel{1} = \cancel{x^2}+\cancel{2x}+\cancel{1}-2x^3-\cancel{4x^2}-\cancel{2x}$

$4x^3=0$

$x=0$ accett. $S=\{0\}$

C.A. $\begin{cases} 1-x>0 \\ 1+x>0 \\ 1+2x>0 \\ 1-2x>0 \end{cases}$ $\begin{cases} x<1 \\ x>-1 \\ x>-\frac{1}{2} \\ x<\frac{1}{2} \end{cases}$

$-\frac{1}{2}<x<\frac{1}{2}$

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$\log_3 x + \log_3(x+1) = \log_3(x^2-x)+1$

$\log_3(x^2+x) = \log_3(3x^2-3x)$

$x^2+x = 3x^2-3x$

$2x^2-4x=0$

$2x(x-2)=0$ $\quad x_1=0$ non accett. $\quad x_2=2$ accett. $\quad S=\{2\}$

C.A. $\begin{cases} x>0 \\ x+1>0 \\ x(x-1)>0 \end{cases} \rightarrow \begin{cases} x>0 \\ x>-1 \\ x<0 \vee x>1 \end{cases} \rightarrow x>1$

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$\frac{1}{2}\log(x+8) - \log 12 = 2\log 5 - 2$ $\quad$ C.A. $x+8>0 \rightarrow x>-8$

$\frac{1}{2}\log(x+8) = \log 12 + \log 25 - \log 100$

$\frac{1}{2}\log(x+8) = \log \frac{12\cdot \cancel{25}^{1}}{\cancel{100}_{4}}$

$\log(x+8) = 2\log 3$

$x+8=9 \rightarrow x=1$ accett. $\quad S=\{1\}$

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$\frac{1}{5}\log(x+2) = 3+\log 0{,}002$ $\quad$ C.A. $x>-2$

$\frac{1}{5}\log(x+2) = \log 1000 + \log 0{,}002$

$\frac{1}{5}\log(x+2) = \log 2$

$\log(x+2) = \log 32$

$x+2=32$

$x=30$ $\quad S=\{30\}$

$\log x + \log(x+3) = 1$ $\quad$ C.A. $\begin{cases} x>0 \\ x+3>0 \end{cases}$ $\begin{cases} x>0 \\ x>-3 \end{cases} \rightarrow x>0$

$\log(x^2+3x) = \log 10$

$x^2+3x-10=0$

$x_{1,2} = \frac{-3\pm 7}{2} = \begin{cases} -5 & \text{non accett.} \\ 2 & \text{accett.} \end{cases}$

$S=\{2\}$

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$\frac{1}{2}\log(9-x) = \log 3 + \frac{1}{2}\log x$ $\quad$ C.A. $\begin{cases} 9-x>0 \\ x>0 \end{cases} \rightarrow \begin{cases} x<9 \\ x>0 \end{cases} \rightarrow 0<x<9$

$\log(9-x) = \log 9 + \log x$

$\log(9-x) = \log 9x$

$9-x=9x$

$10x=9 \rightarrow x=\frac{9}{10}$ accett. $\quad S=\left\{\frac{9}{10}\right\}$

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$\frac{1}{2}\log(1+5x) = \log(1+\sqrt{3x})$

C.A. $\begin{cases} 1+5x>0 \\ 1+\sqrt{3x}>0 \end{cases} \rightarrow \begin{cases} x>-\frac{1}{5} \\ x\geq 0 \end{cases} \rightarrow x\geq 0$

$1+5x = 1+3x+2\sqrt{3x}$

$2x = 2\sqrt{3x}$

$x=\sqrt{3x}$

$x^2=3x$

$x(x-3)=0$

$x_1=0$ accett.

$x_2=3$ accett. $\quad S=\{0;3\}$

$\frac{1}{4}\log_a(x^4+2x-6) = \log_a x$

$\log_a(x^4+2x-6) = \log_a x^4$

$x^4+2x-6 = x^4$

$2x-6=0$

$x=3$ accett.

$\frac{1}{4}\log_a 81 = \log_a 3$

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$\frac{1}{2}\log(x+20) = \log 2 + \frac{1}{4}\log(x+20)$ C.A. $x+20>0$, $x>-20$

$2\log(x+20) = \log 16 + \log(x+20)$

$\log(x+20) = \log 16$

$x+20=16$

$x=-4$ accett. $S=\{-4\}$

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$\log x + \log(2x-1) = \log(2x+5) + \log 3$ C.A. $\begin{cases} x>0 \\ 2x-1>0 \\ 2x+5>0 \end{cases}$ $\begin{cases} x>0 \\ x>\frac{1}{2} \\ x>-\frac{5}{2} \end{cases} \to x>\frac{1}{2}$

$\log(2x^2-x) = \log(6x+15)$

$2x^2-x = 6x+15$

$2x^2-7x-15=0$

$x_{1,2} = \frac{7\pm13}{4} = \begin{cases} -\frac{3}{2} & \text{non accett.} \\ 5 & \text{accett.} \end{cases}$ $S=\{5\}$

n. 38 pag. 473

$\frac{3}{4}\log 2 + \frac{1}{4}\log(5x+2) = \log 4$ C.A. $5x+2>0 \to x>-\frac{2}{5}$

$\log 8 + \log(5x+2) = \log 256$

$\log(5x+2) = \log 32$

$5x+2=32 \to 5x=30 \to x=6$ accett. $S=\{6\}$

n. 39 pag. 473

$\log x - \frac{1}{2}\log(x^2+1) = \log(4-x) - \frac{1}{2}\log(x^2-8x+17)$ C.A. $\begin{cases} x>0 \\ x^2+1>0 \\ 4-x>0 \\ x^2-8x+17>0 \end{cases}$ $\begin{cases} x>0 \\ \forall x\in\mathbb{R} \\ x<4 \\ \forall x\in\mathbb{R} \end{cases}$ $0<x<4$

$\log x^2 + \log(x^2-8x+17) = \log(4-x)^2 + \log(x^2+1)$

$x^4-8x^3+17x^2 = (16-8x+x^2)(x^2+1)$

$\cancel{x^4}\cancel{-8x^3}\cancel{+17x^2} = \cancel{x^4}\cancel{-8x^3}+\cancel{16x^2}\cancel{+x^2}-8x+16$

$8x=16$

$x=2$ accett. $S=\{2\}$

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$\log(2x+11) - \log(x+4) - \log 2 = \log(1-3x) - \log(1-x)$ C.A. $\begin{cases} 2x+11>0 \\ x+4>0 \\ 1-3x>0 \\ 1-x>0 \end{cases}$ $\begin{cases} x>-\frac{11}{2} \\ x>-4 \\ x<\frac{1}{3} \\ x<1 \end{cases}$ $-4<x<\frac{1}{3}$

$\log(2x+11) + \log(1-x) = \log(1-3x) + \log(x+4) + \log 2$

$\cancel{2x}+11-2x^2-11x = \cancel{2x}-6x^2+8-24x$

$4x^2+13x+3=0$

$\Delta = 169-48=121=11^2$

$x_{1,2} = \frac{-13\pm11}{8} = \begin{cases} -3 & \text{accett.} \\ -\frac{1}{4} & \text{accett.} \end{cases}$

$S=\{-3; -\frac{1}{4}\}$

n. 41 p. 474

$$\log\left(1-\sqrt{x+2}\right) = \frac{1}{2}\log(3x+7)$$

$$2\log\left(1-\sqrt{x+2}\right) = \log(3x+7)$$

$$\log\left(1-\sqrt{x+2}\right)^2 = \log(3x+7)$$

$$1+x+2-2\sqrt{x+2} = 3x+7$$

$$-2\sqrt{x+2} = 2x+4$$

$$\sqrt{x+2} = -x-2$$

$$\begin{cases} -x-2 \geq 0 \\ x+2 = x^2+4x+4 \end{cases}$$

$$\begin{cases} x \leq -2 \\ x^2+3x+2=0 \end{cases}$$

$$\begin{cases} x \leq -2 \\ x_{1,2} = \dfrac{-3\pm 1}{2} = \begin{cases} -2 & \text{accett.} \\ -1 & \text{non accett.} \end{cases} \end{cases}$$

$x=-2$ accett.

C.A.

$$\begin{cases} 1-\sqrt{x+2} > 0 \\ 3x+7>0 \end{cases}$$

1ª diseq.

$1-\sqrt{x+2} > 0$

$\sqrt{x+2} < 1$

$$\begin{cases} x+2 \geq 0 \\ x+2<1 \end{cases}$$

$$\begin{cases} x \geq -2 \\ x<-1 \end{cases}$$

$-2 \leq x < -1$

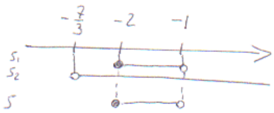
2ª diseq.

$3x+7>0$

$3x>-7$

$x>-\frac{7}{3}$

$$\begin{cases} -2 \leq x < -1 \\ x > -\frac{7}{3} \end{cases}$$



$-2 \leq x < 1$

n. 42 p. 474

$$\frac{1}{2}\log_a(16-x) = \log_a(x-4)$$

$$\log_a(16-x) = 2\log_a(x-4)$$

$$\log_a(16-x) = \log_a(x-4)^2$$

$$16-x = (x-4)^2$$

$$16-x = x^2-8x+16$$

$$x^2-7x=0$$

$$x(x-7)=0$$

$x_1=0$ non accettabile

$x_2=7$ accett.

C.A. $a>0 \wedge a \neq 1$

$$\begin{cases} 16-x>0 \\ x-4>0 \end{cases} \rightarrow \begin{cases} x<16 \\ x>4 \end{cases}$$

$4<x<16$

n. 43 p. 474

$$1 + \log(x+1) = \log 15$$

C.A. $x+1>0$

$x>-1$

$$\log 10 + \log(x+1) = \log 15$$

$$\log(10x+10) = \log 15$$

$$10x+10 = 15$$

$$10x = 5$$

$x = \frac{1}{2}$ accett.

$$\log_3(x-2) + 2 = \log_3 6x$$

C.A.

$$\begin{cases} x-2>0 \\ 6x>0 \end{cases} \rightarrow \begin{cases} x>2 \\ x>0 \end{cases} \rightarrow x>2$$

$$\log_3(x-2) + \log_3 9 = \log_3 6x$$

$$\log_3(9x-18) = \log_3 6x$$

$$9x-18 = 6x$$

$$3x = 18$$

$x = 6$ accett.

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$$\log(x+\sqrt{x}) - \log(x-\sqrt{x}) = \log 6 - \frac{1}{2}\log x$$

$$\log(x+\sqrt{x}) + \frac{1}{2}\log x = \log 6 + \log(x-\sqrt{x})$$

$$\log(x+\sqrt{x}) + \log\sqrt{x} = \log(6x-6\sqrt{x})$$

$$\log(x\sqrt{x}+x) = \log(6x-6\sqrt{x})$$

$$x\sqrt{x}+x = 6x-6\sqrt{x}$$

$$\sqrt{x}(x+6) = 5x$$

$$x+6 = 5\sqrt{x}$$

$$x^2+12x+36 = 25x$$

$$x^2-13x+36 = 0$$

$$(x-9)(x-4) = 0$$

$x_1 = 9$ accett.

$x_2 = 4$ accett.

C.A.

$$\begin{cases} x+\sqrt{x}>0 \\ x-\sqrt{x}>0 \\ x>0 \end{cases}$$

1^a diseq.

$$x+\sqrt{x}>0$$

$$\sqrt{x}>-x$$

$$\begin{cases} -x<0 \\ x\geq 0 \end{cases} \vee \begin{cases} -x\geq 0 \\ x>x^2 \end{cases}$$

$$\begin{cases} x>0 \\ x\geq 0 \end{cases} \vee \begin{cases} x\leq 0 \\ x^2-x<0 \end{cases}$$

$$x>0 \vee \begin{cases} x\leq 0 \\ 0<x<1 \end{cases}$$

$$x>0 \vee \emptyset$$

$$x>0$$

2^a diseq.

$$x-\sqrt{x}>0$$

$$\sqrt{x}<x$$

$$\begin{cases} x\geq 0 \\ x>0 \\ x<x^2 \end{cases} \rightarrow \begin{cases} x\geq 0 \\ x>0 \\ x^2-x>0 \end{cases}$$

$$\begin{cases} x\geq 0 \\ x>0 \\ x<0 \vee x>1 \end{cases} \rightarrow x>1$$

C.A. $\begin{cases} x>0 \\ x>1 \\ x>0 \end{cases} \rightarrow x>1$

n. 45 p. 474

$$\frac{1}{2}\left[\log\left(\sqrt{2}\,x-1\right)+\log\left(\sqrt{2}\,x+1\right)\right]=\log\left(x^2+1\right)-\frac{1}{2}\left[\log\left(x-1\right)+\log\left(x+1\right)\right]$$

C.A. $\begin{cases}\sqrt{2}\,x-1>0\\ \sqrt{2}\,x+1>0\\ x^2+1>0\\ x-1>0\\ x+1>0\end{cases} \rightarrow \begin{cases}x>\frac{1}{\sqrt{2}}\\ x>-\frac{1}{\sqrt{2}}\\ \forall x\in\mathbb{R}\\ x>1\\ x>-1\end{cases} \rightarrow x>1$

$$\frac{1}{2}\log\left(2x^2-1\right)=\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2-1\right)$$

$$\frac{1}{2}\log\left(2x^2-1\right)+\frac{1}{2}\log\left(x^2-1\right)=\log\left(x^2+1\right)$$

$$\log\left(2x^2-1\right)+\log\left(x^2-1\right)=2\log\left(x^2+1\right)$$

$$\log\left(2x^4-3x^2+1\right)=\log\left(x^2+1\right)^2$$

$$2x^4-3x^2+1=x^4+2x^2+1$$

$$x^4-5x^2=0$$

$$x^2\left(x^2-5\right)=0$$

$x_1=x_2=0$ non accett.

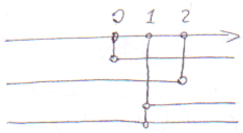
$x_3=-\sqrt{5}$ non accett.

$x_4=\sqrt{5}$ accett.

n. 46 p. 474

$$\log 2+\frac{1}{2}\left[\log x+\log\left(2-x\right)\right]-2\log\left(x-1\right)=\frac{1}{2}\log 3-\log\left(1-x\right)^2$$

C.A. $\begin{cases}x>0\\ 2-x>0\\ x-1>0\\ (1-x)^2>0\end{cases} \rightarrow \begin{cases}x>0\\ x<2\\ x>1\\ \forall x\neq 1\end{cases}$



$1<x<2$

$$\log 2+\frac{1}{2}\log\left(2x-x^2\right)\cancel{-2\log\left(x-1\right)}=\frac{1}{2}\log 3\cancel{-2\log\left(x-1\right)}$$

$(1-x)^2=(x-1)^2$

$\log(1-x)^2=$

$=\log(x-1)^2=$

$=2\log(x-1)$

$$2\log 2+\log\left(2x-x^2\right)=\log 3$$

$$\log 4+\log\left(2x-x^2\right)=\log 3$$

$$\log\left(8x-4x^2\right)=\log 3$$

$$8x-4x^2=3$$

$$4x^2-8x+3=0$$

$$x_{1,2}=\frac{4\pm\sqrt{4}}{4}=\begin{cases}\overset{-}{\ }\ \frac{1}{2} & \text{non accett.}\\ \overset{+}{\ }\ \frac{3}{2} & \text{accett.}\end{cases}$$

n. 47 p. 474

$$\log\left(\sqrt{1+x}+1\right)=\frac{1}{2}\log\left(1+x+\sqrt{1-x}\right)$$

C.A. $\begin{cases}\sqrt{1+x}+1>0\\ 1+x+\sqrt{1-x}>0\end{cases}$

1ª diseq.

$\sqrt{1+x}+1>0$

$\sqrt{1+x}>-1$

$x+1\geqslant 0$

$\boxed{x\geqslant -1}$

2ª diseq.

$1+x+\sqrt{1-x}>0$

$\sqrt{1-x}>-x-1$

$\begin{cases}-x-1<0\\ 1-x\geqslant 0\end{cases} \vee \begin{cases}-1-x\geqslant 0\\ 1-x>x^2+2x+1\end{cases}$

$\begin{cases}x>-1\\ x\leqslant 1\end{cases} \vee \begin{cases}x\leqslant -1\\ x^2+3x<0\end{cases}$

$\begin{cases}x>-1\\ x\leqslant 1\end{cases} \vee \begin{cases}x\leqslant -1\\ -3<x<0\end{cases}$

$-1<x\leqslant 1 \quad\vee\quad -3<x\leqslant -1$

$\boxed{-3<x\leqslant 1}$

C.A. $\begin{cases}x\geqslant -1\\ -3<x\leqslant 1\end{cases} \rightarrow \quad -1\leqslant x\leqslant 1$

$$2\log\left(\sqrt{1+x}+1\right)=\log\left(1+x+\sqrt{1-x}\right)$$

$$\log\left(\sqrt{1+x}+1\right)^2=\log\left(1+x+\sqrt{1-x}\right)$$

$$\cancel{1+x}+1+2\sqrt{1+x}=\cancel{1+x}+\sqrt{1-x}$$

$$\left(1+2\sqrt{1+x}\right)^2=\left(\sqrt{1-x}\right)^2$$

$$1+4+4x+4\sqrt{1+x}=1-x$$

$$4\sqrt{1+x}=-5x-4$$

$\begin{cases}-5x-4\geqslant 0\\ \cancel{16}+16x=25x^2+40x+\cancel{16}\end{cases}$

$\begin{cases}x\leqslant -\frac{4}{5}\\ 25x^2+24x=0\end{cases}$

$\begin{cases}x\leqslant -\frac{4}{5}\\ x(25x+24)=0\end{cases}$

$x_1=0$ non accett.

$x_2=-\frac{24}{25}$ accett.

n. 48 p. 474

$$\frac{1}{2}\left[\log(2x-3)-\log(x-2)\right]=\log\left(2+\sqrt{x-2}\right)-\log 2-\frac{1}{2}\log(x-2)$$

C.A. $\begin{cases} 2x-3>0 \\ x-2>0 \\ 2+\sqrt{x-2}>0 \\ x-2>0 \end{cases} \rightarrow \begin{cases} x>\frac{3}{2} \\ x>2 \\ x>2 \\ x>2 \end{cases} \rightarrow x>2$

$$\frac{1}{2}\log(2x-3)\cancel{-\frac{1}{2}\log(x-2)}=\log\left(2+\sqrt{x-2}\right)-\log 2\cancel{-\frac{1}{2}\log(x-2)}$$

$$\log(2x-3)=2\log\left(2+\sqrt{x-2}\right)-2\log 2$$

$$\log(2x-3)=\log\left(2+\sqrt{x-2}\right)^2-\log 4$$

$$\log(2x-3)+\log 4=\log\left(4+x-2+4\sqrt{x-2}\right)$$

$$\log(8x-12)=\log\left(x+2+4\sqrt{x-2}\right)$$

$$8x-12=x+2+4\sqrt{x-2}$$

$$7x-14=4\sqrt{x-2}$$

$$7(x-2)=4\sqrt{x-2}$$

$$\sqrt{x-2}=\frac{4}{7}$$

$$x-2=\frac{16}{49}$$

$$x=2+\frac{16}{49}$$

$$x=\frac{98+16}{49}$$

$$x=\frac{114}{49}$$ accettabile

n. 49 p. 474

$$\frac{1}{2}\log_3\left(3x+\sqrt{6x-1}\right)=\log_3(2x+1)-\frac{1}{2}\log_3\left(3x-\sqrt{6x-1}\right)$$

C.A. $\begin{cases}3x+\sqrt{6x-1}>0\\2x+1>0\end{cases}$

1ª diseq.

$3x+\sqrt{6x-1}>0$

$\sqrt{6x-1}>-3x$

$\begin{cases}-3x<0\\6x-1\geq 0\end{cases} \vee \begin{cases}-3x\geq 0\\6x-1>9x^2\end{cases}$

$\begin{cases}x>0\\x\geq\frac{1}{6}\end{cases} \vee \begin{cases}x\leq 0\\9x^2-6x+1<0\end{cases}$

$\begin{cases}x>0\\x\geq\frac{1}{6}\end{cases} \vee \begin{cases}x\leq 0\\(3x-1)^2<0\end{cases}$

$x\geq\frac{1}{6} \quad \vee \quad \emptyset$

$x\geq\frac{1}{6}$

2ª diseq.

$2x+1>0$

$2x>-1$

$x>-\frac{1}{2}$

C.A. $\begin{cases}x\geq\frac{1}{6}\\x>-\frac{1}{2}\end{cases} \qquad x\geq\frac{1}{6}$

$$\frac{1}{2}\log_3\left(3x+\sqrt{6x-1}\right)+\frac{1}{2}\log_3\left(3x-\sqrt{6x-1}\right)=\log_3(2x+1)$$

$$\log_3\left(3x+\sqrt{6x-1}\right)+\log_3\left(3x-\sqrt{6x-1}\right)=2\log_3(2x+1)$$

$$\log_3\left(9x^2-6x+1\right)=2\log_3(2x+1)$$

$$\log_3(3x-1)^2=2\log_3(2x+1)$$

$$\cancel{2}\log(3x-1)=\cancel{2}\log_3(2x+1)$$

$$3x-1=2x+1$$

$x=2$ accettabile

n. 50 p. 474

$$\log\left(\sqrt{x+2}-\sqrt{x-2}\right)-\log\left(\sqrt{x+2}+\sqrt{x-2}\right)=\log x-\log 2$$

C. A. $\begin{cases}\sqrt{x+2}-\sqrt{x-2}>0\\ \sqrt{x+2}+\sqrt{x-2}>0\\ x>0\end{cases}$

1ª diseq.

$\sqrt{x+2}-\sqrt{x-2}>0$

$\sqrt{x+2}>\sqrt{x-2}$

$\begin{cases}x+2\geq 0\\ x-2\geq 0\\ x+2>x-2\end{cases}$ $\begin{cases}x\geq -2\\ x\geq 2\\ \forall x\in\mathbb{R}\end{cases}$

$x\geq 2$

2ª diseq.

$\sqrt{x+2}+\sqrt{x-2}>0$

$\begin{cases}x+2\geq 0\\ x-2\geq 0\end{cases}$

$\begin{cases}x\geq -2\\ x\geq 2\end{cases}$

$x\geq 2$

C. A. $\begin{cases}x\geq 2\\ x\geq 2\\ x>0\end{cases} \longrightarrow x\geq 2$

$$\log\left(\sqrt{x+2}-\sqrt{x-2}\right)+\log 2=\log x+\log\left(\sqrt{x+2}+\sqrt{x-2}\right)$$

$$\log\left(2\sqrt{x+2}-2\sqrt{x-2}\right)=\log\left(x\sqrt{x+2}+x\sqrt{x-2}\right)$$

$$2\sqrt{x+2}-2\sqrt{x-2}=x\sqrt{x+2}+x\sqrt{x-2}$$

$$2\sqrt{x+2}-x\sqrt{x+2}=x\sqrt{x-2}+2\sqrt{x-2}$$

$$(2-x)\sqrt{x+2}=(x+2)\sqrt{x-2}$$

$$2-x=\sqrt{x+2}\,\sqrt{x-2}$$

$$2-x=\sqrt{x^2-4}$$

$$\begin{cases}2-x\geq 0\\ 4-4x+x^2=x^2-4\end{cases}$$

$$\begin{cases}x\leq 2\\ 4x=8\end{cases}$$

$x=2$ accettabile

n. 51 p. 474

$$\log_5 7 - \log_5 \left(3\sqrt{x+3}+1\right) = 1 - \log_5 \left(3\sqrt{x+3}-1\right)$$

C.A. $\begin{cases} 3\sqrt{x+3}+1>0 \\ 3\sqrt{x+3}-1>0 \end{cases}$

1ª diseq.	2ª diseq.
$3\sqrt{x+3}+1>0$	$3\sqrt{x+3}-1>0$
$3\sqrt{x+3}>-1$	$3\sqrt{x+3}>1$
$x+3\geq 0$	$9x+27>1$
$x\geq -3$	$9x>-26$
	$x>-\frac{26}{9}$

C.A. $\begin{cases} x\geq -3 \\ x>-\frac{26}{9} \end{cases} \rightarrow x>-\frac{26}{9}$

$$\log_5 7 + \log_5 \left(3\sqrt{x+3}-1\right) = \log_5 5 + \log_5 \left(3\sqrt{x+3}+1\right)$$

$$\log_5 \left(21\sqrt{x+3}-7\right) = \log_5 \left(15\sqrt{x+3}+5\right)$$

$$21\sqrt{x+3}-7 = 15\sqrt{x+3}+5$$

$$6\sqrt{x+3} = 12$$

$$\sqrt{x+3} = 2$$

$$x+3 = 4$$

$x = 1$ accettabile

n. 52 p. 474

$$\frac{1}{2}\log_2 (3x+2) = \log_2 x + \frac{1}{2}$$

C.A. $\begin{cases} 3x+2>0 \\ x>0 \end{cases}$ $\begin{cases} x>-\frac{2}{3} \\ x>0 \end{cases}$ $x>0$

$$\log_2 (3x+2) = 2\log_2 x + 1$$

$$\log_2 (3x+2) = \log_2 x^2 + \log_2 2$$

$$\log_2 (3x+2) = \log_2 2x^2$$

$$3x+2 = 2x^2$$

$$2x^2-3x-2=0$$

$x_{1,2} = \frac{3\pm 5}{4} = \begin{cases} - & -\frac{1}{2} \quad \text{non accett.} \\ + & 2 \quad \text{accett.} \end{cases}$

n. 53 p. 474

$$\log\left(\sqrt{2x+3}-1\right)-\log 2=\log 3-\frac{1}{2}\log(2x+3)$$

C.A. $\begin{cases}\sqrt{2x+3}-1>0\\ 2x+3>0\end{cases} \rightarrow \begin{cases}x>-1\\ x>-\frac{3}{2}\end{cases} \rightarrow x>-1$

1ª diseq.

$\sqrt{2x+3}-1>0$

$\sqrt{2x+3}>1$

$2x+3>1$

$x>-1$

2ª diseq.

$2x+3>0$

$2x>-3$

$x>-\frac{3}{2}$

$$\log\left(\sqrt{2x+3}-1\right)=\log 2+\log 3-\log\sqrt{2x+3}$$

$t=\sqrt{2x+3}$

$\log(t-1)+\log t=\log 6$ C.A. $\begin{cases}t>0\\ t-1>0\end{cases} \rightarrow t>1$

$\log(t^2-t)=\log 6$

$t^2-t=6$

$t^2-t-6=0$ $t_{1,2}=\frac{1\pm5}{2}=\begin{cases}-2 & \text{non accett.}\\ 3 & \text{accett.}\end{cases}$

$t=3$

$\sqrt{2x+3}=3$

$2x+3=9 \rightarrow 2x=6 \rightarrow x=3$ accett.

n. 54 p. 474

$\log x+\log\frac{x+3}{10}=0$

C.A. $\begin{cases}x>0\\ \frac{x+3}{10}>0\end{cases} \rightarrow \begin{cases}x>0\\ x>-3\end{cases} \rightarrow x>0$

$\log\frac{x^2+3x}{10}=\log 1$

$\frac{x^2+3x}{10}=1$

$x^2+3x=10$

$x^2+3x-10=0$

$(x+5)(x-2)=0$

$x_1=-5$ non accett.

$x_2=2$ accett.

$\frac{1}{2}\log_8\left(x+\sqrt{x-4}\right)=\frac{1}{3}$

C.A. $x+\sqrt{x-4}>0$

$\sqrt{x-4}>-x$

$\begin{cases}-x<0\\ x-4\geq 0\end{cases} \vee \begin{cases}-x\geq 0\\ x-4>x^2\end{cases}$

$\begin{cases}x>0\\ x\geq 4\end{cases} \vee \begin{cases}x\leq 0\\ x^2-x+4<0\end{cases}$

$x\geq 4 \vee \emptyset$

$x\geq 4$

$\log_8\left(x+\sqrt{x-4}\right)=\frac{2}{3}=\frac{2}{3}\log_8 8=\log_8 8^{\frac{2}{3}}$

$x+\sqrt{x-4}=4$

$\sqrt{x-4}=4-x$

$\begin{cases}4-x\geq 0\\ x-4=16-8x+x^2\end{cases} \quad \begin{cases}x\leq 4\\ x^2-9x+20=0\end{cases}$

$x_{1,2}=\frac{9\pm1}{2}=\begin{cases}4 & \text{accett.}\\ 5 & \text{non accett.}\end{cases}$

n. 55 p. 474

$$\log_2(x-2)+\log_2(6-x)=2$$

C.A. $\begin{cases} x-2>0 \\ 6-x>0 \end{cases} \rightarrow \begin{cases} x>2 \\ x<6 \end{cases} \rightarrow 2<x<6$

$$\log_2(-x^2+8x-12)=\log_2 4$$

$$-x^2+8x-12=4$$

$$x^2-8x+16=0$$

$$(x-4)^2=0$$

$x=4$ accett.

$$1+\log x = 2\log 5$$

C.A. $x>0$

$$\log 10+\log x=\log 5^2$$

$$\log 10x=\log 25$$

$$10x=25$$

$x=\frac{5}{2}$ accett.

n. 56 p. 474

$$\log_3 x-\log_3(2x-1)=2$$

C.A. $\begin{cases} x>0 \\ 2x-1>0 \end{cases}$

$\begin{cases} x>0 \\ x>\frac{1}{2} \end{cases} \rightarrow x>\frac{1}{2}$

$$\log_3 x=\log_3 9+\log_3(2x-1)$$

$$\log_3 x=\log_3(18x-9)$$

$$x=18x-9$$

$$17x=9$$

$x=\frac{9}{17}$ accett.

$$\log(x-1)-\log 2x=\log 5$$

C.A. $\begin{cases} x-1>0 \\ 2x>0 \end{cases} \rightarrow \begin{cases} x>1 \\ x>0 \end{cases} \rightarrow x>1$

$$\log(x-1)=\log 5+\log 2x$$

$$\log(x-1)=\log 10x$$

$$x-1=10x$$

$$9x=-1$$

$x=-\frac{1}{9}$ non accett.

L'eq. è impossibile

$S=\emptyset$

n. 57 p. 474

$$\log_2(x+2)+\log_2 x=3$$

C.A. $\begin{cases} x+2>0 \\ x>0 \end{cases} \rightarrow \begin{cases} x>-2 \\ x>0 \end{cases} \rightarrow x>0$

$$\log_2(x^2+2x)=\log_2 8$$

$$x^2+2x=8$$

$$x^2+2x-8=0$$

$x_{1,2}=-1\pm 3 = \begin{cases} -4 & \text{non accett.} \\ 2 & \text{accett.} \end{cases}$

$$\log(3x-8)+\log(x-3)=\log 2$$

C.A. $\begin{cases} 3x-8>0 \\ x-3>0 \end{cases} \rightarrow \begin{cases} x>\frac{8}{3} \\ x>3 \end{cases} \rightarrow x>3$

$$\log(3x^2-17x+24)=\log 2$$

$$3x^2-17x+24=2$$

$$3x^2-17x+22=0$$

$x_{1,2}=\frac{17\pm\sqrt{25}}{6}=\frac{17\pm5}{6}= \begin{cases} - \quad 2 & \text{non accett.} \\ + \quad \frac{11}{3} & \text{accett.} \end{cases}$

n. 58 p. 474

$$\log(9-x) + \log(x-4) - \log 4 = 0$$

C.A. $\begin{cases} 9-x>0 \\ x-4>0 \end{cases} \rightarrow \begin{cases} x<9 \\ x>4 \end{cases} \rightarrow 4<x<9$

$$\log(-x^2+13x-36) = \log 4$$

$$-x^2+13x-36 = 4$$

$$x^2-13x+40=0$$

$$x_{1,2} = \frac{13 \pm \sqrt{169-160}}{2} = \frac{13 \pm 3}{2}$$

$x_{1,2} = \Big\langle \begin{matrix} 5 \\ 8 \end{matrix}$ $\begin{matrix} \text{accett.} \\ \text{accett.} \end{matrix}$

$$2\log(x+1) = \log(x+1) + \log 6$$

C.A. $x+1>0$

$x>-1$

$$\log(x+1) = \log 6$$

$$x+1 = 6$$

$x = 5$ accett.

n. 59 p. 475

$$\log(x^2-x+2) - \log(2x-3) = \log(x+2)$$

$$\log(x^2-x+2) = \log(x+2) + \log(2x-3)$$

$$\log(x^2-x+2) = \log(2x^2+x-6)$$

$$x^2-x+2 = 2x^2+x-6$$

$$x^2+2x-8=0$$

$x_{1,2} = -1 \pm 3 = \Big\langle \begin{matrix} -4 \\ 2 \end{matrix}$ $\begin{matrix} \text{non accett.} \\ \text{accett.} \end{matrix}$

C.A.

$\begin{cases} x^2-x+2>0 \\ 2x-3>0 \\ x+2>0 \end{cases}$

$\begin{cases} \forall x \in \mathbb{R} \\ x>\frac{3}{2} \\ x>-2 \end{cases} \rightarrow x>\frac{3}{2}$

$$\log(x-1) = 3$$

C.A. $x-1>0 \rightarrow x>1$

$$\log(x-1) = 3\log e$$

$$\log(x-1) = \log e^3$$

$$x-1 = e^3$$

$x = e^3+1$ accett.

n. 60 p. 475

$\log_3 |3-2x| = 2$

C.A. $|3-2x|>0$

$3-2x \neq 0$

$x \neq \frac{3}{2}$

$\log_3 |3-2x| = \log_3 9$

$|3-2x| = 9$

$3-2x=-9 \quad \vee \quad 3-2x=9$

$2x=12 \quad \vee \quad 2x=-6$

$x=6 \quad \vee \quad x=-3$

accett. accett.

$\log_2 |2x^2-x| + \log_2 \frac{1}{3} = 0$

C.A. $|2x^2-x|>0$

$2x^2-x \neq 0$

$x(2x-1) \neq 0$

$x \neq 0 \quad \wedge \quad x \neq \frac{1}{2}$

$\log_2 |2x^2-x| = -\log_2 \frac{1}{3}$

$\log_2 |2x^2-x| = \log_2 3$

$|2x^2-x| = 3$

$2x^2-x=-3 \quad \vee \quad 2x^2-x=3$

$2x^2-x+3=0 \quad \vee \quad 2x^2-x-3=0$

$\Delta = 1-24<0 \quad \vee \quad x_{1,2} = \frac{1 \pm 5}{4}$

$\emptyset$

$x_1 = -1$ accett.

$x_2 = \frac{3}{2}$ accett.

n. 61 p. 475

$\log_4 (2x+3) + \log_{2x+3} 4 = \frac{5}{2}$

C.A.

$\begin{cases} 2x+3>0 \\ 2x+3 \neq 1 \end{cases} \rightarrow \begin{cases} x > -\frac{3}{2} \\ x \neq -1 \end{cases}$

$-\frac{3}{2} < x < -1 \quad \vee \quad x > -1$

$\log_4 (2x+3) + \frac{\log_4 4}{\log_4 (2x+3)} = \frac{5}{2}$

$t = \log_4 (2x+3)$

$t + \frac{1}{t} = \frac{5}{2} \qquad t \neq 0$

$t^2+1 = \frac{5}{2}t$

$2t^2+2 = 5t$

$2t^2-5t+2=0 \longrightarrow t_{1,2} = \frac{5 \pm 3}{4} = \begin{cases} - & \frac{1}{2} \\ + & 2 \end{cases}$

$\log_4 (2x+3) = \frac{1}{2} \quad \vee \quad \log_4 (2x+3) = 2$

$\log_4 (2x+3) = \log_4 2 \quad \vee \quad \log_4 (2x+3) = \log_4 16$

$2x+3=2 \quad \vee \quad 2x+3=16$

$2x=-1 \quad \vee \quad 2x=13$

$x=-\frac{1}{2} \quad \vee \quad x=\frac{13}{2}$

accett. accett.

n. 77 pag. 476

$\log^2 x + \log x - 6 = 0$

C.A. $x>0$

$t = \log x$

$t^2 + t - 6 = 0$

$t_{1,2} = \frac{-1 \pm 5}{2} = \left\langle \begin{array}{l} -3 \\ 2 \end{array} \right.$

$\log x = -3 \rightarrow \log x = \log e^{-3} \rightarrow x = e^{-3}$

$\log x = 2 \rightarrow \log x = \log e^2 \rightarrow x = e^2$

$S = \{ e^{-3} ; e^2 \}$

$2 \log_2^2 x - 17 \log_2 x + 8 = 0$

C.A. $x>0$

$t = \log_2 x$

$2t^2 - 17t + 8 = 0$

$t_{1,2} = \frac{17 \pm 15}{4} = \left\langle \begin{array}{l} \frac{1}{2} \\ 8 \end{array} \right.$

$\log_2 x = \frac{1}{2} \rightarrow x = \sqrt{2}$

$\log_2 x = 8 \rightarrow x = 2^8$

$S = \{ \sqrt{2} ; 2^8 \}$

n. 78 pag. 476

$$\frac{6}{(\log x)^2 - 1} + \frac{3}{\log x + 1} = \frac{\log x + 1}{\log x - 1}$$

C.A. $x>0$

$t = \log x$

$t \neq 1$

$t \neq -1$

$$\frac{6}{t^2-1} + \frac{3}{t+1} = \frac{t+1}{t-1}$$

$$\frac{6 + 3t - 3}{(t+1)(t-1)} = \frac{t^2 + 2t + 1}{(t+1)(t-1)}$$

$3 + 3t = t^2 + 2t + 1$

$t^2 - t - 2 = 0$

$(t+1)(t-2) = 0$

$t_1 = -1$ non accett.

$t_2 = 2$ $\log_{10} x = 2 \rightarrow x = 10^2 \rightarrow x = 100$ $S = \{100\}$

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$\log_2 a - 2 = \frac{1}{2} \log_2 (9x^2 + a^2) - \log_4 25$ $a>0$

$\log_2 a^2 - 4 = \log_2 (9x^2 + a^2) - 2\log_2 5$

$\log_2 \frac{25a^2}{16} = \log_2 (9x^2 + a^2)$

$9x^2 + a^2 = \frac{25}{16} a^2$

$9x^2 = \frac{9}{16} a^2 \rightarrow x^2 = \frac{a^2}{16} \rightarrow x = \pm \frac{a}{4}$ $S = \{ -\frac{a}{4} ; +\frac{a}{4} \}$

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$\log_3^2 x + 2 \log_3 x - 3 = 0$

C.A. $x>0$

$t = \log_3 x$

$t^2 + 2t - 3 = 0$

$(t+3)(t-1) = 0$

$t_1 = -3$ $\log_3 x = -3 \rightarrow x = 3^{-3} \rightarrow x = \frac{1}{27}$

$t_2 = 1$ $\log_3 x = 1 \rightarrow x = 3$

$S = \{ \frac{1}{27} ; 3 \}$

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$$2\log^2_{\frac{1}{2}} x - 5\log_{\frac{1}{2}} x + 2 = 0$$

C.A. $x>0$

$t = \log_{\frac{1}{2}} x$

$2t^2 - 5t + 2 = 0$

$t_{1,2} = \frac{5 \pm 3}{4} = \begin{cases} \frac{1}{2} \\ 2 \end{cases}$

$\log_{\frac{1}{2}} x = \frac{1}{2} \rightarrow x = \sqrt{\frac{1}{2}}$

$\log_{\frac{1}{2}} x = 2 \rightarrow x = \frac{1}{4}$

$S = \left\{ \frac{1}{4} ; \sqrt{\frac{1}{2}} \right\}$

$$3\log^2_{\frac{1}{3}} x - 2\log_{\frac{1}{3}} x - 1 = 0$$

C.A. $x>0$

$t = \log_{\frac{1}{3}} x$

$3t^2 - 2t - 1 = 0$

$t_{1,2} = \frac{1 \pm 2}{3} = \begin{cases} -\frac{1}{3} \\ 1 \end{cases}$

$\log_{\frac{1}{3}} x = -\frac{1}{3} \rightarrow x = \left(\frac{1}{3}\right)^{-\frac{1}{3}} \rightarrow x = \sqrt[3]{3}$

$\log_{\frac{1}{3}} x = 1 \rightarrow x = \frac{1}{3}$

$S = \left\{ \frac{1}{3} ; \sqrt[3]{3} \right\}$

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$$\log^2_3 x + \log_3 x - 6 = 0$$

C.A. $x>0$

$t = \log_3 x$

$t^2 + t - 6 = 0$

$(t+3)(t-2) = 0$

$t_1 = -3 \rightarrow \log_3 x = -3 \rightarrow x = \frac{1}{27}$

$t_2 = 2 \rightarrow \log_3 x = 2 \rightarrow x = 9$

$S = \left\{ \frac{1}{27} ; 9 \right\}$

$$3\log^2_2 x + 5\log_2 x - 2 = 0$$

C.A. $x>0$

$t = \log_2 x$

$3t^2 + 5t - 2 = 0$

$t_{1,2} = \frac{-5 \pm 7}{6} = \begin{cases} -2 \\ \frac{1}{3} \end{cases}$

$\log_2 x = -2 \rightarrow x = \frac{1}{4}$

$\log_2 x = \frac{1}{3} \rightarrow x = \sqrt[3]{2}$

$S = \left\{ \frac{1}{4} ; \sqrt[3]{2} \right\}$

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$$\frac{5}{\log x + 4} - \frac{3}{\log x - 2} = 4$$

C.A. $x>0$

$t = \log x$

$t \neq -4$

$t \neq 2$

$\frac{5}{t+4} - \frac{3}{t-2} = 4$

$5t - 10 - 3t - 12 = 4t^2 + 8t - 32$

$4t^2 + 6t - 10 = 0$

$2t^2 + 3t - 5 = 0$

$t_1 = 1 \rightarrow \log x = 1 \rightarrow x = 10$

$t_2 = -\frac{5}{2} \rightarrow \log x = -\frac{5}{2} \rightarrow x = 10^{-\frac{5}{2}}$

$S = \left\{ 10^{-\frac{5}{2}} ; 10 \right\}$

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$$\frac{\log x - 4}{\log x + 1} = \frac{15}{4} + \frac{\log x + 1}{\log x - 4}$$

C.A. $x>0$

$t = \log x$

$t \neq -1$

$t \neq 4$

$\frac{t-4}{t+1} = \frac{15}{4} + \frac{t+1}{t-4}$

$\frac{4(t^2-8t+16)}{\cancel{4(t+1)(t-4)}} = \frac{15(t^2-3t-4)+4(t^2+2t+1)}{\cancel{4(t+1)(t-4)}}$

$\cancel{4t^2} - 32t + 64 = 15t^2 - 45t - 60 + \cancel{4t^2} + 8t + 4$

$15t^2 - 5t - 120 = 0$

$3t^2 - t - 24 = 0 \qquad t_{1,2} = \frac{1 \pm 17}{6} = \begin{cases} -\frac{8}{3} \\ 3 \end{cases}$

$\log x = -\frac{8}{3} \rightarrow x = 10^{-\frac{8}{3}}$

$\log x = 3 \rightarrow x = 10^3$

$S = \left\{ 10^3 ; 10^{-\frac{8}{3}} \right\}$

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$$\frac{2\log_4 x-5}{2\log_4 x+1}+\frac{6}{\log_4 x^2-3}=\frac{7}{4}$$

C.A. $x>0$

$t=\log_4 x$

$$\frac{2t-5}{2t+1}+\frac{6}{2t-3}=\frac{7}{4} \qquad t\neq -\frac{1}{2} \qquad t\neq \frac{3}{2}$$

$$\frac{(8t-12)(2t-5)+24(2t+1)}{\cancel{4(2t+1)(2t-3)}}=\frac{7(4t^2-4t-3)}{\cancel{4(2t+1)(2t-3)}}$$

$16t^2-40t-24t+60+48t+24=28t^2-28t-21$

$16t^2-16t+84=28t^2-28t-21$

$12t^2-12t-105=0$

$4t^2-4t-35=0 \qquad t_{1,2}=\frac{2\pm12}{4}=\begin{cases}-\frac{5}{2}\\ \frac{7}{2}\end{cases}$

$\log_4 x=-\frac{5}{2} \rightarrow x=4^{-\frac{5}{2}} \rightarrow x=\frac{1}{32}$

$\log_4 x=\frac{7}{2} \rightarrow x=4^{\frac{7}{2}} \rightarrow x=2^7$

$S=\left\{\frac{1}{32};2^7\right\}$

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$\log(4^x-3)-2\log 3=(1-2x)\log 2$

C.A. $4^x-3>0$

$4^x>3$

$x\log 4>\log 3 \rightarrow x>\frac{\log 3}{\log 4}$

$\log(4^x-3)=\log 2^{1-2x}+\log 9$

$\log(4^x-3)=\log(9\cdot 2^{1-2x})$

$2^{2x}-3=18\cdot\frac{1}{2^{2x}} \qquad t=2^{2x}$

$t^2-3t-18=0$

$t_{1,2}=\frac{3\pm9}{2}=\begin{cases}-3\\ 6\end{cases}$

$2^{2x}=-3$ impossibile

$2^{2x}=6 \rightarrow x\log 4=\log 6 \rightarrow x=\frac{\log 6}{\log 4}$ accettabile

$S=\left\{\frac{\log 6}{\log 4}\right\}$

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$3\left(\log_2 x+\log_{\frac{1}{2}} x\right)=4-\log_2\frac{1}{x}$ C.A. $x>0$

$3\left(\log_2 x-\log_2 x\right)=4+\log_2 x$

$\log_2 x=-4 \rightarrow x=2^{-4} \rightarrow x=\frac{1}{16}$ $\qquad S=\left\{\frac{1}{16}\right\}$

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$\log_3(x+3)+\log_{\frac{1}{3}}(x^2+x-2)=\log_{\frac{1}{3}}(x+2)+\log_3 2$

C.A. $\begin{cases}x+3>0\\ x^2+x-2>0\\ x+2>0\end{cases}$

$\begin{cases}x>-3\\ x<-2 \vee x>1\\ x>-2\end{cases}$

$x>1$

$\log_3(x+3)-\log_3(x^2+x-2)=-\log_3(x+2)+\log_3 2$

$\log_3(x+3)+\log_3(x+2)=\log_3(x^2+x-2)+\log_3 2$

$\log_3(x^2+5x+6)=\log_3(2x^2+2x-4)$

$x^2+5x+6=2x^2+2x-4$

$x^2-3x-10=0$

$x_{1,2}=\frac{3\pm7}{2}=\begin{cases}-2 & \text{non accett.}\\ 5 & \text{accett.}\end{cases}$

$S=\{5\}$

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$(2+x)\log 4 = \frac{10}{x}\log 8$ C.A. $x \neq 0$

$(4+2x)\cancel{\log 2} = \frac{30}{x}\cancel{\log 2}$

$2x^2+4x-30=0$

$x^2+2x-15=0$ $x_{1,2} = -1 \pm 4 = \left\langle \begin{matrix} -5 \\ +3 \end{matrix} \right.$ $S=\{-5;3\}$

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$x\log 3 + \log(1+3^x) = 1+2\log 3$

$\log 3^x + \log(1+3^x) = \log 10 + \log 9$

$\log(3^x+3^{2x}) = \log 90$

$3^{2x}+3^x-90=0$ $3^x = \frac{-1\pm 19}{2} = \left\langle \begin{matrix} -10 \\ 9 \end{matrix} \right.$ $3^x=-10$ impossibile

$3^x = 9 \rightarrow x=2$ $S=\{2\}$

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$1 + x\log 3 = 2x\log 2 + \log 5 - \log 16$

$\log 10 - \log 5 + \log 2^4 = x(\log 4 - \log 3)$

$x = \frac{5\log 2}{\log 4 - \log 3}$

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$(2x+1)\log 2 = \log(1+2^x)$

$\log 2^{2x+1} = \log(1+2^x)$

$2^{2x+1} = 1+2^x$ $t=2^x$

$2t^2-t-1=0$

$t_{1,2} = \frac{1\pm 3}{4} = \left\langle \begin{matrix} -\frac{1}{2} \\ 1 \end{matrix} \right.$

$2^x = -\frac{1}{2}$ impossibile

$2^x = 1 \rightarrow x=0$

$S=\{0\}$

$\log_3(2-3^x)+x=0$ C.A.

$2-3^x>0$

$3^x<2$

$x<\frac{\log 2}{\log 3}$

$\log_3(2-3^x) = -x\log_3 3$

$\log_3(2-3^x) = \log_3 3^{-x}$

$2-3^x = \frac{1}{3^x}$ $t=3^x$

$2-t = \frac{1}{t}$

$-t^2+2t-1=0$

$(t-1)^2=0$

$t=1$

$3^x=1 \rightarrow x=0$ accett.

$S=\{0\}$

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$(2+x)\log 4 = \frac{10}{x}\log 8$ C.A. $x \neq 0$

$(4+2x)\cancel{\log 2} = \frac{30}{x}\cancel{\log 2}$

$2x^2+4x-30=0$

$x^2+2x-15=0$ $x_{1,2} = -1 \pm 4 = \begin{cases} -5 \\ +3 \end{cases}$ $S=\{-5;3\}$

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$x\log 3 + \log(1+3^x) = 1+2\log 3$

$\log 3^x + \log(1+3^x) = \log 10 + \log 9$

$\log(3^x+3^{2x}) = \log 90$

$3^{2x}+3^x-90=0$ $3^x = \frac{-1\pm 19}{2} = \begin{cases} -10 \\ 9 \end{cases}$ $3^x=-10$ impossibile

$3^x=9 \rightarrow x=2$ $S=\{2\}$

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$1+x\log 3 = 2x\log 2 + \log 5 - \log 16$

$\log 10 - \log 5 + \log 2^4 = x(\log 4 - \log 3)$

$x = \frac{5\log 2}{\log 4 - \log 3}$

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$(2x+1)\log 2 = \log(1+2^x)$

$\log 2^{2x+1} = \log(1+2^x)$

$2^{2x+1} = 1+2^x$ $t=2^x$

$2t^2-t-1=0$

$t_{1,2} = \frac{1\pm 3}{4} = \begin{cases} -\frac{1}{2} \\ 1 \end{cases}$

$2^x = -\frac{1}{2}$ impossibile

$2^x = 1 \rightarrow x=0$

$S=\{0\}$

$\log_3(2-3^x)+x=0$ C.A.

$2-3^x>0$

$3^x<2$

$x<\frac{\log 2}{\log 3}$

$\log_3(2-3^x) = -x\log_3 3$

$\log_3(2-3^x) = \log_3 3^{-x}$

$2-3^x = \frac{1}{3^x}$ $t=3^x$

$2-t = \frac{1}{t}$

$-t^2+2t-1=0$

$(t-1)^2=0$

$t=1$

$3^x=1 \rightarrow x=0$ accett.

$S=\{0\}$

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$x \operatorname{Log} 3 = \operatorname{Log}\left(2-3^{-x}\right)$

C.A. $2-3^{-x}>0$

$2\cdot 3^{x}-1>0$

$3^{x}>\frac{1}{2}$

$x>-\frac{\operatorname{Log}2}{\operatorname{Log}3}$

$\operatorname{Log} 3^{x} = \operatorname{Log}\left(2-3^{-x}\right)$

$3^{x} = 2-\frac{1}{3^{x}}$ $\qquad t=3^{x}$

$t = 2-\frac{1}{t}$

$t^2-2t+1=0 \rightarrow (t-1)^2=0 \rightarrow t=1 \rightarrow 3^{x}=1 \rightarrow x=0$ accett. $\qquad S=\{0\}$

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$\operatorname{Log}\left(4^{1-x}+2\right)-\operatorname{Log}2 = \operatorname{Log}\left(2^{2x+1}-3\right)$

C.A.

$2^{2x+1}-3>0$

$2^{2x+1}>3$

$2x+1>\frac{\operatorname{Log}3}{\operatorname{Log}2}$

$x>\frac{\operatorname{Log}3}{2\operatorname{Log}2}-\frac{1}{2} \approx 0{,}292481$

$\operatorname{Log}\left(\frac{4}{4^{x}}+2\right) = \operatorname{Log}2+\operatorname{Log}\left(2\cdot 4^{x}-3\right)$

$\frac{4}{4^{x}}+2 = 4\cdot 4^{x}-6$ $\qquad t=4^{x}$

$\frac{4}{t}+2 = 4t-6$

$4t-8-\frac{4}{t}=0$

$t^2-2t-1=0$ $\qquad t_{1,2}=1\pm\sqrt{2}$

$4^{x}=1-\sqrt{2}$ impossibile

$4^{x}=1+\sqrt{2} \rightarrow x=\frac{\operatorname{Log}(1+\sqrt{2})}{\operatorname{Log}4} \approx 0{,}635776$ accett.

$S=\left\{\frac{\operatorname{Log}(1+\sqrt{2})}{\operatorname{Log}4}\right\}$

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$\operatorname{Log}\left(2^{x-1}+3^{x+2}\right) = \operatorname{Log}3+\operatorname{Log}\left(3^{x-2}+\sqrt{4^{x+1}}\right)$

$\operatorname{Log}\left(\frac{1}{2}\cdot 2^{x}+9\cdot 3^{x}\right) = \operatorname{Log}\left(\frac{1}{3}\cdot 3^{x}+6\cdot 2^{x}\right)$

$\frac{1}{2}\cdot 2^{x}+9\cdot 3^{x} = \frac{1}{3}\cdot 3^{x}+6\cdot 2^{x}$

$\left(9-\frac{1}{3}\right)\cdot 3^{x} = \left(6-\frac{1}{2}\right)\cdot 2^{x}$

$\frac{26}{3}\cdot 3^{x} = \frac{11}{2}\cdot 2^{x}$

$\left(\frac{3}{2}\right)^{x} = \frac{33}{52}$

$\operatorname{Log}\left(\frac{3}{2}\right)^{x} = \operatorname{Log}\frac{33}{52}$

$x\operatorname{Log}\left(\frac{3}{2}\right) = \operatorname{Log}\frac{33}{52}$

$x = \frac{\operatorname{Log}33-\operatorname{Log}52}{\operatorname{Log}3-\operatorname{Log}2}$ $\qquad S=\left\{\frac{\operatorname{Log}33-\operatorname{Log}52}{\operatorname{Log}3-\operatorname{Log}2}\right\}$

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$$\log_3\left(\sqrt{3^x}-8\sqrt[4]{3^x}\right)=2 \qquad t=\sqrt[4]{3^x}$$

$$\log_3\left(t^2-8t\right)=\log_3 9 \qquad \text{C.A.} \quad t^2-8t>0$$

$$t<0 \lor t>8$$

$$t^2-8t-9=0$$

$$t_{1,2}=4\pm 5=\left\langle\begin{matrix}-1\\ 9\end{matrix}\right.$$

$t=-1 \rightarrow \sqrt[4]{3^x}=-1$ impossibile

$t=9 \rightarrow \sqrt[4]{3^x}=9 \rightarrow 3^{\frac{x}{4}}=3^2 \rightarrow \frac{x}{4}=2 \rightarrow x=8 \qquad S=\{8\}$

$$\frac{\log\left(2\cdot 3^{\frac{1}{x}}+3\cdot 3^{\frac{1}{2x}}\right)}{\log\left(1+2\cdot 9^{\frac{1}{4x}}\right)}=1 \qquad t=3^{\frac{1}{2x}}$$

$$\log\left(2t^2+3t\right)=\log\left(1+2t\right)$$

$$2t^2+3t=1+2t$$

$$2t^2+t-1=0 \qquad t_{1,2}=\frac{-1\pm 3}{4}=\left\langle\begin{matrix}-1\\ \frac{1}{2}\end{matrix}\right.$$

$3^{\frac{1}{2x}}=-1$ impossibile

$3^{\frac{1}{2x}}=\frac{1}{2} \rightarrow \frac{1}{2x}\log 3=-\log 2 \rightarrow x=-\frac{\log 3}{2\log 2}$

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$$\left(\log_3 x-\frac{1}{2}\right)^2+2\log_3 x=3\log_3 x^2-\frac{23}{4} \qquad \text{C.A.} \quad x>0$$

$$t=\log_3 x$$

$$\left(t-\frac{1}{2}\right)^2+2t=6t-\frac{23}{4}$$

$$t^2-t+\frac{1}{4}+2t=6t-\frac{23}{4}$$

$$t^2-5t+6=0$$

$$t_{1,2}=\frac{5\pm 1}{2}=\left\langle\begin{matrix}2\\ 3\end{matrix}\right.$$

$\log_3 x=2 \rightarrow x=3^2 \rightarrow x=9$

$\log_3 x=3 \rightarrow x=3^3 \rightarrow x=27 \qquad S=\{9;27\}$

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$$\frac{\log\sqrt{x}+2}{3\log\sqrt{x}} = 2-\log\sqrt{x} \qquad C.A. \quad x>0$$

$t = \log\sqrt{x}$

$$\frac{t+2}{3t} = 2-t \qquad t\neq 0$$

$t+2 = 6t-3t^2$

$3t^2-5t+2=0 \qquad t_{1,2} = \frac{5\pm 1}{6} = \left\langle \begin{array}{l} \frac{2}{3} \\ 1 \end{array}\right.$

$\log\sqrt{x} = \frac{2}{3} \rightarrow \sqrt{x} = 10^{\frac{2}{3}} \rightarrow x = 10^{\frac{4}{3}}$

$\log\sqrt{x} = 1 \rightarrow \sqrt{x} = 10 \rightarrow x = 10^2 \qquad S=\left\{10^{\frac{4}{3}}\,;\,10^2\right\}$

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$$\frac{\log x+1}{\log x-1} + \frac{\log x-1}{\log x+1} = \frac{2}{\log^2 x-1} \qquad C.A. \quad x>0$$

$t = \log x$

$$\frac{t+1}{t-1} + \frac{t-1}{t+1} = \frac{2}{t^2-1} \qquad t\neq 1\ ,\ t\neq -1$$

$$\frac{t^2+\cancel{2t}+1+t^2-\cancel{2t}+1}{(t-1)(t+1)} = \frac{2}{(t+1)(t-1)}$$

$2t^2=0 \rightarrow t=0 \rightarrow \log x = 0 \rightarrow x=1 \qquad S=\{1\}$

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$$\frac{\log_a x}{1-\log_a x^3} = 1+\frac{\log_a x+1}{\log_a x-1}\ , \quad \text{con } a\in\mathbb{R}^+-\{1\} \qquad C.A. \quad x>0$$

$t = \log_a x$

$$\frac{t}{1-3t} = 1+\frac{t+1}{t-1} \qquad t\neq 1,\quad t\neq\frac{1}{3}$$

$t^2-t = t-3t^2\cancel{-1}\cancel{+3t}+t\cancel{+1}-3t^2\cancel{-3t}$

$7t^2-3t=0$

$t=0 \rightarrow \log_a x = 0 \rightarrow x = a^0 \rightarrow x=1$

$t=\frac{3}{7} \rightarrow \log_a x = \frac{3}{7} \rightarrow x = a^{\frac{3}{7}} \rightarrow x=\sqrt[7]{a^3}$

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$$\left(2-\log_{\frac{1}{2}}\sqrt[3]{x}\right)\left(2+\log_{\frac{1}{2}}\sqrt[3]{x}\right)=\log_{\frac{1}{2}}x \qquad C.A. \quad x>0$$

$$\left(2-\tfrac{1}{3}\log_{\frac{1}{2}}x\right)\left(2+\tfrac{1}{3}\log_{\frac{1}{2}}x\right)=\log_{\frac{1}{2}}x$$

$$t=\log_{\frac{1}{2}}x$$

$$\left(2-\tfrac{1}{3}t\right)\left(2+\tfrac{1}{3}t\right)=t$$

$$4-\tfrac{1}{9}t^2=t$$

$$t^2+9t-36=0 \qquad t_{1,2}=\frac{-9\pm15}{2}=\begin{cases}-12\\3\end{cases}$$

$$\log_{\frac{1}{2}}x=-12 \rightarrow x=\left(\tfrac{1}{2}\right)^{-12} \rightarrow x=2^{12}$$

$$\log_{\frac{1}{2}}x=3 \rightarrow x=\left(\tfrac{1}{2}\right)^{3} \rightarrow x=\tfrac{1}{8} \qquad S=\left\{\tfrac{1}{8};\,2^{12}\right\}$$

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$$\left(\log_2 x-1\right)\left(\log_2\sqrt{x}-\log_2 x+1\right)+6=0 \qquad C.A. \quad x>0$$

$$\left(\log_2 x-1\right)\left(\tfrac{1}{2}\log_2 x-\log_2 x+1\right)+6=0$$

$$t=\log_2 x$$

$$(t-1)\left(1-\tfrac{1}{2}t\right)+6=0$$

$$-\tfrac{1}{2}t^2+\tfrac{1}{2}t+t-1+6=0$$

$$t^2-3t-10=0 \qquad t_{1,2}=\frac{3\pm7}{2}=\begin{cases}-2\\5\end{cases}$$

$$\log_2 x=-2 \rightarrow x=2^{-2} \rightarrow x=\tfrac{1}{4}$$

$$\log_2 x=5 \rightarrow x=2^{5} \rightarrow x=32 \qquad S=\left\{\tfrac{1}{4};\,32\right\}$$

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$$\log\left(7^x+\frac{1}{7^{1-x}}\right)=\log 2+\log\left(3\cdot5^x+5^{x+2}\right)$$

$$\log\left(7^x+\tfrac{1}{7}\cdot7^x\right)=\log 2+\log\left(28\cdot5^x\right)$$

$$\log\left(\tfrac{8}{7}\cdot7^x\right)=\log\left(56\cdot5^x\right)$$

$$\tfrac{8}{7}\cdot7^x=56\cdot5^x$$

$$\left(\tfrac{7}{5}\right)^x=49$$

$$x\log\tfrac{7}{5}=\log 49$$

$$x=\frac{\log 49}{\log 7-\log 5} \qquad S=\left\{\frac{\log 49}{\log 7-\log 5}\right\}$$

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$$x \log 3 + \log(2\cdot 3^x - 3) + \log 5 = 3\log 3 + \log(2 - 3^{x-2})$$

$$\log\left[3^x(2\cdot 3^x - 3)\cdot 5\right] = \log\left[27\left(2 - \frac{1}{9}\cdot 3^x\right)\right]$$

C.A.

$$\begin{cases} 2\cdot 3^x - 3 > 0 \\ 2 - 3^{x-2} > 0 \end{cases}$$

$$\begin{cases} 3^x > \frac{3}{2} \\ \frac{1}{9}\cdot 3^x < 2 \end{cases}$$

$$\begin{cases} x > \frac{\log 3 - \log 2}{\log 3} \\ x < \frac{\log 18}{\log 3} \end{cases}$$

$$t = 3^x$$

$$5(2t^2 - 3t) = 54 - 3t$$

$$10t^2 - 12t - 54 = 0$$

$$5t^2 - 6t - 27 = 0 \qquad t_{1,2} = \frac{3 \pm 12}{5} = \begin{cases} -\frac{9}{5} \\ 3 \end{cases}$$

$t = -\frac{9}{5} \rightarrow 3^x = -\frac{9}{5}$ impossibile

$t = 3 \rightarrow 3^x = 3 \rightarrow x = 1$ accett. $\qquad S = \{1\}$

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$$\log(3^{1+x} + 2^{2-x}) = \log 15 - x\log 2$$

$$\log\left(3\cdot 3^x + \frac{4}{2^x}\right) = \log\left(15\cdot\frac{1}{2^x}\right)$$

$$3\cdot 3^x + \frac{4}{2^x} = \frac{15}{2^x}$$

$$3\cdot 3^x = \frac{11}{2^x}$$

$$6^x = \frac{11}{3} \rightarrow \log 6^x = \log\frac{11}{3} \rightarrow x = \frac{\log 11 - \log 3}{\log 6} \qquad S = \left\{\frac{\log 11 - \log 3}{\log 6}\right\}$$

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$$\frac{\log_2(4^{x+1} - 2) - 2x}{2x+1} = 1$$

C.A.

$$\begin{cases} 4^{x+1} - 2 > 0 \\ 2x + 1 \neq 0 \end{cases}$$

$$\begin{cases} 2^{2x+2} > 2 \\ x \neq -\frac{1}{2} \end{cases}$$

$$x > -\frac{1}{2}$$

$$\log_2(4^{x+1} - 2) - 2x = 2x + 1$$

$$\log_2(2^{2x+2} - 2) = (4x+1)\log_2 2$$

$$\log_2(2^{2x+2} - 2) = \log_2 2^{4x+1}$$

$$4\cdot 2^{2x} - 2 = 2^{4x}\cdot 2 \qquad t = 2^{2x} = 4^x$$

$$2t^2 - 4t + 2 = 0$$

$$t^2 - 2t + 1 = 0$$

$$(t-1)^2 = 0$$

$t = 1 \rightarrow 4^x = 1 \rightarrow x = 0$ accett. $\qquad S = \{0\}$

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$x^{\log x} = 10$ C.A. $x>0$

$\log x^{\log x} = 1$

$\log^2 x = 1$

$\log x = \left\langle \begin{matrix} +1 \\ -1 \end{matrix} \right.$

$\log x = 1 \rightarrow x = 10$

$\log x = -1 \rightarrow x = \frac{1}{10}$

$S = \left\{ \frac{1}{10} ; 10 \right\}$

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$x^{\log \sqrt{x}} = 100$ C.A. $x>0$

$\log \sqrt{x} \log x = 2$

$\frac{1}{2} \log^2 x = 2$

$\log^2 x = 4$ $\log x = \left\langle \begin{matrix} +2 \\ -2 \end{matrix} \right.$

$\log x = 2 \rightarrow x = 10^2$

$\log x = -2 \rightarrow x = 10^{-2}$

$S = \left\{ \frac{1}{100} ; 100 \right\}$

$\frac{10}{x^{\log x}} = \frac{\sqrt{10}}{x^{\log \sqrt{x}}}$ C.A. $x>0$

$\frac{10}{\sqrt{10}} = x^{\log x - \frac{1}{2}\log x}$

$\sqrt{10} = x^{\frac{1}{2}\log x}$

$\frac{1}{2} = \frac{1}{2} \log x \cdot \log x$

$\log^2 x = 1$ $\log x = \pm 1$

$x = \left\langle \begin{matrix} \frac{1}{10} \\ 10 \end{matrix} \right.$ $S = \left\{ \frac{1}{10} ; 10 \right\}$

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$\frac{1}{2} \log_3 \left(10 - \frac{3}{3^{2x-3}} \right) = x-1$

$\log_3 \left(10 - \frac{81}{3^{2x}} \right) = (2x-2) \log_3 3$

$\log_3 \left(10 - \frac{81}{3^{2x}} \right) = \log_3 3^{2x-2}$

$10 - \frac{81}{3^{2x}} = \frac{1}{9} \cdot 3^{2x}$ $t = 3^{2x}$

$10 - \frac{81}{t} = \frac{1}{9} t$

$90t - 729 = t^2$

$t^2 - 90t + 729 = 0$

$t_{1,2} = 45 \pm 36 = \left\langle \begin{matrix} 9 \\ 81 \end{matrix} \right.$

$3^{2x} = 9 \rightarrow 2x = 2 \rightarrow x = 1 \rightarrow \frac{1}{2} \log_3 1 = 0$ accett.

$3^{2x} = 81 \rightarrow 2x = 4 \rightarrow x = 2 \rightarrow \frac{1}{2} \log_3 9 = 1$ accett.

$S = \{ 1 ; 2 \}$