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$$\frac{3^{1-x}\cdot 9^{2+x}}{27^{x}} = \frac{1}{3}$$

$$\frac{3^{1-x}\cdot 3^{4+2x}}{3^{3x}} = 3^{-1}$$

$$3^{5+x-3x} = 3^{-1}$$

$$5-2x = -1$$

$$-2x = -6$$

$$x = 3$$

$$\left(\frac{2}{3}\right)^{x+1} = \left(\frac{27}{8}\right)^{1-2x}$$

$$\left(\frac{2}{3}\right)^{x+1} = \left(\frac{3}{2}\right)^{3-6x}$$

$$\left(\frac{2}{3}\right)^{x+1} = \left(\frac{2}{3}\right)^{6x-3}$$

$$x+1 = 6x-3$$

$$-5x = -4$$

$$x = \frac{4}{5}$$

$$\sqrt{2\sqrt{4^{x}}} = 4$$

$$\sqrt{\sqrt{4^{x+1}}} = 4$$

$$4^{\frac{x+1}{4}} = 4$$

$$\frac{x+1}{4} = 1$$

$$x+1 = 4$$

$$x = 3$$

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$$\sqrt[x-1]{125}\cdot\sqrt[x+2]{5^{-8}} = \sqrt[x-1]{25}\cdot\sqrt[2x-1]{5^{9}} \qquad x>1,\ x\in\mathbb{N}$$

$$5^{\frac{3}{x-1}}\cdot 5^{\frac{8}{x+2}} = 5^{\frac{2}{x-1}}\cdot 5^{\frac{9}{2x-1}}$$

$$5^{\frac{3}{x-1}+\frac{8}{x+2}} = 5^{\frac{2}{x-1}+\frac{9}{2x-1}}$$

$$\frac{3x+6+8x-8}{(x-1)(x+2)} = \frac{4x-2+9x-9}{(x-1)(2x-1)}$$

$$\frac{11x-2}{x+2} = \frac{13x-11}{2x-1}$$

$$22x^2-4x-11x+2 = 13x^2-11x+26x-22$$

$$9x^2-30x+24 = 0$$

$$\frac{\Delta}{4} = 225-216 = 9$$

$$x_{1,2} = \frac{15\pm 3}{9} = \begin{cases} \frac{12}{9} & \text{non accett.} \\ 2 & \text{accett.} \end{cases}$$

$$\frac{9^{x+1}}{27^{3-2x}} = \frac{1}{81}\cdot 3^{14x}$$

$$\frac{3^{2x+2}}{3^{9-6x}} = \frac{3^{1+x}}{3^{4}}$$

$$3^{8x-7} = 3^{x-3}$$

$$8x-7 = x-3$$

$$7x = 4$$

$$x = \frac{4}{7}$$

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$$\sqrt[1+x]{2^{3x}} = \sqrt[x]{2^{x+2}}\cdot\sqrt[2x]{2^{x-2}} \qquad x>0,\ x\in\mathbb{N}$$

$$2^{\frac{3x}{1+x}} = 2^{\frac{x+2}{x}}\cdot 2^{\frac{x-2}{2x}}$$

$$2^{\frac{3x}{1+x}} = 2^{\frac{x+2}{x}+\frac{x-2}{2x}}$$

$$\frac{3x}{1+x} = \frac{3x+2}{2x}$$

$$6x^2 = 3x^2+2x+3x+2$$

$$3x^2-5x-2 = 0$$

$$\Delta = 25+24 = 49 = 7^2$$

$$x_{1,2} = \frac{5\pm 7}{6} = \begin{cases} -\frac{1}{3} & \text{non accett.} \\ 2 & \text{accett.} \end{cases}$$

$$x = 2$$

$$\frac{\left(3^{x+1}\right)^{2x-1}\cdot 27^{1-x}}{9^{2-x}} = 1$$

$$\frac{3^{2x^2+x-1}\cdot 3^{3-3x}}{3^{4-2x}} = 3^{0}$$

$$3^{2x^2-2x+2-4+2x} = 3^{0}$$

$$2x^2-2 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

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$$\sqrt[1+x^2]{8^{5x^2-3}} = 4^{\frac{3(5-x^2)}{2(3x^2+1)}}$$

$$2^{\frac{3(5x^2-3)}{1+x^2}} = 2^{\frac{3(5-x^2)}{2(3x^2+1)}}$$

$$\frac{3(5x^2-3)}{1+x^2} = \frac{3(5-x^2)}{3x^2+1}$$

$$15x^4 - 9x^2 + 5x^2 - 3 = 5x^2 - x^4 + 5 - x^2$$

$$16x^4 - 8x^2 - 8 = 0$$

$$2x^4 - x^2 - 1 = 0$$

$$x^2 = \frac{1 \pm 3}{4} = \begin{cases} -\frac{1}{2} \\ 1 \end{cases}$$

$x^2 = -\frac{1}{2}$ impossibile

$x^2 = 1 \rightarrow x = \pm 1$

$$\frac{\sqrt[3]{32^x}}{\left(2^{x+2}\right)^{x-2}} = 1$$

$$\frac{2^{\frac{5}{3}x}}{2^{x^2-4}} = 2^0$$

$$2^{-x^2+\frac{5}{3}x+4} = 2^0$$

$$x^2 - \frac{5}{3}x - 4 = 0$$

$$3x^2 - 5x - 12 = 0$$

$$\Delta = 25 + 144 = 169 = 13^2$$

$$x_{1,2} = \frac{5 \pm 13}{6} = \begin{cases} -\frac{4}{3} \\ 3 \end{cases}$$

$$\frac{\sqrt[3]{4^{x+1}} \cdot 2^{x-1}}{\sqrt{2^x}} = \frac{1}{4}$$

$$\frac{2^{\frac{2x+2}{3}} \cdot 2^{x-1}}{2^{\frac{x}{2}}} = 2^{-2}$$

$$2^{\left(\frac{2x+2}{3} + x - 1 - \frac{x}{2}\right)} = 2^{-2}$$

$$\frac{2x+2}{3} + x - 1 - \frac{x}{2} = -2$$

$$4x + 4 + 6x - 6 - 3x = -12$$

$$7x = -10$$

$$x = -\frac{10}{7}$$

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$$\frac{\sqrt[2x-12]{4^{3x}}}{\sqrt{2^{x-1}}} = \frac{4}{\sqrt[x-6]{8^{6+x}}} \qquad x > 6 \quad x \in \mathbb{N}$$

$$\frac{2^{\frac{3x}{2(x-6)}}}{2^{\frac{x-1}{2}}} = \frac{2^2}{2^{\frac{18+3x}{x-6}}}$$

$$2^{\frac{3x}{x-6}} \cdot 2^{\frac{18+3x}{x-6}} = 2^2 \cdot 2^{\frac{x-1}{2}}$$

$$\frac{3x}{x-6} + \frac{18+3x}{x-6} = 2 + \frac{x-1}{2}$$

$$\frac{18+6x}{x-6} = \frac{3+x}{2}$$

$$36 + 12x = x^2 - 3x - 18$$

$$x^2 - 15x - 54 = 0$$

$$(x-18)(x+3) = 0$$

$x = -3$ non accett.

$x = 18$ accett.

$$\sqrt[x+2]{\left(\frac{2}{3}\right)^{4x}} = \frac{16}{81}\sqrt[x]{\left(\frac{3}{2}\right)^{x+2}} \qquad x > 0 \quad x \in \mathbb{N}$$

$$\left(\frac{2}{3}\right)^{\frac{4x}{x+2}} = \left(\frac{2}{3}\right)^4 \left(\frac{3}{2}\right)^{\frac{x+2}{x}}$$

$$\frac{4x}{x+2} = 4 - \frac{x+2}{x}$$

$$\frac{4x}{x+2} = \frac{3x-2}{x}$$

$$4x^2 = 3x^2 + 4x - 4$$

$$x^2 - 4x + 4 = 0$$

$$(x-2)^2 = 0$$

$$x = 2$$

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$$3^{x-2} \cdot 5^{x-2} = 1$$

$$15^{x-2} = 15^0$$

$$x - 2 = 0$$

$$x = 2$$

$$3^x = 2^x$$

$$\left(\frac{3}{2}\right)^x = 1$$

$$\left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^0$$

$$x = 0$$

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$$2^x \cdot 5^{2x-3} = \frac{1}{2^{x-3}}$$

$$2^x \cdot 2^{x-3} \cdot 5^{2x-3} = 1$$

$$2^{2x-3} \cdot 5^{2x-3} = 1$$

$$10^{2x-3} = 10^0 \quad \rightarrow \quad 2x-3=0 \quad \rightarrow \quad x = \frac{3}{2}$$

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$$2^{2x+4} \cdot 3^x = \frac{2}{3^{x+3}}$$

$$\frac{2^{2x+4} \cdot 3^x \cdot 3^{x+3}}{2} = 1$$

$$2^{2x+3} \cdot 3^{2x+3} = 1$$

$$6^{2x+3} = 6^0 \quad \rightarrow \quad 2x+3=0 \quad \rightarrow \quad x = -\frac{3}{2}$$

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$$\frac{2^x \cdot 15}{2^3+1} = 40 \cdot 3^{x-4}$$

$$\frac{2^x \cdot 15^3}{9} = \frac{40^8 \cdot 3^x}{81}$$

$$\frac{2^x}{3^x} = \frac{8}{81} \cdot \frac{9}{3} \quad \rightarrow \quad \left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^3 \quad \rightarrow \quad x = 3$$

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$$8^{x-2} \sqrt[3]{12} = \frac{\sqrt[6]{9^{4+x}}}{32 \sqrt[3]{81^{1-2x}}}$$

$$2^{3x-6} \sqrt[3]{12} = \frac{3^{\frac{4+x}{3}}}{2^5 \cdot 3^{\frac{4-8x}{3}}}$$

$$2^{3x-1} \, 2^{\frac{2}{3}} \cdot 3^{\frac{1}{3}} = 3^{\frac{9x}{3}}$$

$$2^{3x} \cdot 2^{-\frac{1}{3}} \cdot 3^{\frac{1}{3}} = 3^{3x}$$

$$\left(\frac{2}{3}\right)^{3x} = \left(\frac{2}{3}\right)^{\frac{1}{3}} \quad \rightarrow \quad 3x = \frac{1}{3} \quad \rightarrow \quad x = \frac{1}{9}$$

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$$2^x + 2^{x-1} + 2^{x-2} = 7$$

$$2^x + \frac{1}{2} \cdot 2^x + \frac{1}{4} \cdot 2^x = 7$$

$$2^x \left(1 + \frac{1}{2} + \frac{1}{4}\right) = 7$$

$$2^x \cdot \frac{7}{4} = 7 \quad \rightarrow \quad 2^x = 4 \quad \rightarrow \quad 2^x = 2^2 \quad \rightarrow \quad x = 2$$

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$$3^{x+1} - \frac{3^x}{9} + 3^x = 35$$

$$3 \cdot 3^x - \frac{1}{9} \cdot 3^x + 3^x = 35$$

$$3^x \left(3 - \frac{1}{9} + 1\right) = 35$$

$$3^x \cdot \frac{35}{9} = 35 \rightarrow 3^x = 9 \rightarrow 3^x = 3^2 \rightarrow x = 2$$

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$$3^{x+2} + 3^{x+1} + 3^x = 351$$

$$9 \cdot 3^x + 3 \cdot 3^x + 3^x = 351$$

$$13 \cdot 3^x = 351$$

$$3^x = \frac{351}{13}$$

$$3^x = 27 \rightarrow 3^x = 3^3 \rightarrow x = 3$$

$$4^{x-1} + 4^x + 4^{x+1} = \frac{21}{8}$$

$$\frac{1}{4} \cdot 4^x + 4^x + 4 \cdot 4^x = \frac{21}{8}$$

$$4^x \left(\frac{1}{4} + 1 + 4\right) = \frac{21}{8}$$

$$4^x \cdot \frac{21}{4} = \frac{21}{8}$$

$$4^x = \frac{1}{2} \rightarrow 2^{2x} = 2^{-1} \rightarrow 2x = -1 \rightarrow x = -\frac{1}{2}$$

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$$3^{x+1} + 3^{x-2} + 3^{x-1} + 3^{x+2} = 336$$

$$3^x \left(3 + \frac{1}{9} + \frac{1}{3} + 9\right) = 336$$

$$3^x \cdot \frac{112}{9} = 336$$

$$3^x = 27 \rightarrow 3^x = 3^3 \rightarrow x = 3$$

$$2^{2x+1} + 4^{x-1} + 8^{\frac{2}{3}x} = 13$$

$$2 \cdot 2^{2x} + 2^{2x-2} + 2^{2x} = 13$$

$$2^{2x} \left(2 + \frac{1}{4} + 1\right) = 13$$

$$2^{2x} \cdot \frac{13}{4} = 13 \rightarrow 2^{2x} = 4 \rightarrow 2^{2x} = 2^2 \rightarrow x = 1$$

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$$3^{2+\sqrt{x}} + 3^{1+\sqrt{x}} - 3^{\sqrt{x}} = 99$$

$$9 \cdot 3^{\sqrt{x}} + 3 \cdot 3^{\sqrt{x}} - 3^{\sqrt{x}} = 99$$

$$3^{\sqrt{x}} \left(9 + 3 - 1\right) = 99$$

$$3^{\sqrt{x}} \cdot 11 = 99$$

$$3^{\sqrt{x}} = 9 \rightarrow 3^{\sqrt{x}} = 3^2 \rightarrow \sqrt{x} = 2 \rightarrow x = 4$$

$$2^{2x-1} + 2^{2x+1} = 4^x + 6$$

$$\frac{1}{2} \cdot 2^{2x} + 2 \cdot 2^{2x} - 2^{2x} = 6$$

$$2^{2x} \left(\frac{1}{2} + 2 - 1\right) = 6$$

$$2^{2x} \cdot \frac{3}{2} = 6$$

$$2^{2x} = 4 \rightarrow 2^{2x} = 2^2 \rightarrow 2x = 2 \rightarrow x = 1$$

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$$2^x + \frac{2^x}{2} + 2^{x-2} + \frac{2^x}{8} + 2^{x-4} = 62$$

$$2^x \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right) = 62$$

$$2^x \cdot \frac{31}{16} = 62$$

$$2^x = 32 \rightarrow 2^x = 2^5 \rightarrow x = 5$$

$$9^{4x-1} + 2 \cdot 9^{4x+1} - 81^{2x+\frac{3}{2}} + 6398 = 0$$

$$9^{4x} \left(\frac{1}{9} + 18 - 729\right) = -6398$$

$$9^{4x} \left(-\frac{6398}{9}\right) = -6398$$

$$9^{4x} = 9$$

$$4x = 1 \rightarrow x = \frac{1}{4}$$

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$$\left(9^{x-1}\right)^{2x-1} \cdot 3^{x-2} = \frac{1}{3}$$

$$\left(3^{2x-2}\right)^{2x-1} \cdot 3^{x-2} = 3^{-1}$$

$$3^{4x^2-6x+2} \cdot 3^{x-2} = 3^{-1}$$

$$3^{4x^2-5x} = 3^{-1}$$

$$4x^2-5x = -1$$

$$4x^2-5x+1 = 0$$

$$x_{1,2} = \frac{5 \pm 3}{8} = \begin{cases} \frac{1}{4} \\ 1 \end{cases}$$

$$3 \cdot 2^x + 2^{x+3} - 2^{x-1} - 5 \cdot 2^{x+1} = \sqrt{2}$$

$$2^x\left(3 + 8 - \frac{1}{2} - 10\right) = 2^{\frac{1}{2}}$$

$$2^x \cdot \frac{1}{2} = 2^{\frac{1}{2}}$$

$$2^x = 2^{\frac{3}{2}} \rightarrow x = \frac{3}{2}$$

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$$3^x - 3^{x-2} = 2^x + 2^{x+1}$$

$$3^x\left(1 - \frac{1}{9}\right) = 2^x(1+2)$$

$$3^x \cdot \frac{8}{9} = 2^x \cdot 3$$

$$\frac{3^x}{2^x} = \frac{27}{8} \rightarrow \left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^3 \rightarrow x = 3$$

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$$7^{1-x} + 7^{2-x} = 49^{x+\frac{1}{2}} + 49^{x+1}$$

$$7^{-x}\left(7 + 49\right) = 7^{2x}\left(7 + 49\right)$$

$$7^{-x} = 7^{2x}$$

$$-x = 2x \rightarrow x = 0$$

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$$3^{x-1} + 3^{x+1} - 3^{x+2} = 3^{2x-2} - 2 \cdot 9^x$$

$$3^x\left(\frac{1}{3} + 3 - 9\right) = 3^{2x}\left(\frac{1}{9} - 2\right)$$

$$3^x\left(-\frac{17}{3}\right) = 3^{2x}\left(-\frac{17}{9}\right)$$

$$3^x = 3^{2x} \cdot \frac{1}{3}$$

$$3^{x+1} = 3^{2x}$$

$$2x = x+1$$

$$x = 1$$

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$3^{2x} - 3^x - 6 = 0$

$t = 3^x$

$t^2 - t - 6 = 0 \qquad t_{1,2} = \frac{1 \pm 5}{2} = \begin{cases} -2 \\ 3 \end{cases}$

$3^x = -2$ impossibile

$3^x = 3 \rightarrow x = 1$

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$4^x - 6 \cdot 2^x + 8 = 0$

$t = 2^x \qquad t^2 = 2^{2x} = 4^x$

$t^2 - 6t + 8 = 0 \qquad t_{1,2} = 3 \pm 1 = \begin{cases} 2 \\ 4 \end{cases}$

$2^x = 2 \rightarrow x = 1$

$2^x = 4 \rightarrow 2^x = 2^2 \rightarrow x = 2$

$9^x + 6 \cdot 3^x - 27 = 0$

$t = 3^x \qquad t^2 = (3^x)^2 = 3^{2x} = 9^x$

$t^2 + 6t - 27 = 0 \qquad t_{1,2} = -3 \pm 6 = \begin{cases} -9 \\ +3 \end{cases}$

$3^x = -9$ impossibile

$3^x = 3 \rightarrow x = 1$

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$4^x + 2^{x+2} - 12 = 0$

$2^{2x} + 4 \cdot 2^x - 12 = 0$

$t = 2^x \rightarrow t^2 + 4t - 12 = 0 \rightarrow t_{1,2} = -2 \pm 4 = \begin{cases} -6 \\ 2 \end{cases}$

$2^x = -6$ impossibile

$2^x = 2 \rightarrow x = 1$

$3^{2x+1} + 26 \cdot 3^x - 9 = 0$

$t = 3^x \rightarrow 3t^2 + 26t - 9 = 0 \rightarrow t_{1,2} = (-13 \pm 14)/3 = \begin{cases} -9 \\ \frac{1}{3} \end{cases}$

$3^x = -9$ impossibile

$3^x = \frac{1}{3} \rightarrow x = -1$

$2^{2x} + 2^{x+1} - 8 = 0$

$2^{2x} + 2 \cdot 2^x - 8 = 0$

$t = 2^x \rightarrow t^2 + 2t - 8 = 0 \rightarrow t_{1,2} = -1 \pm 3 = \begin{cases} -4 \\ +2 \end{cases}$

$2^x = -4$ impossibile

$2^x = 2 \rightarrow x = 1$

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$12\left(\frac{4}{9}\right)^x - 35\left(\frac{2}{3}\right)^x + 18 = 0$

$t = \left(\frac{2}{3}\right)^x \qquad t^2 = \left[\left(\frac{2}{3}\right)^x\right]^2 = \left(\frac{2}{3}\right)^{2x} = \left(\frac{4}{9}\right)^x$

$12t^2 - 35t + 18 = 0$

$\Delta = 361 = 19^2$

$t_{1,2} = \frac{35 \pm 19}{24} = \begin{cases} \frac{16}{24} = \frac{2}{3} \\ \frac{54}{24} = \frac{9}{4} \end{cases}$

$\left(\frac{2}{3}\right)^x = \frac{2}{3} \rightarrow x = 1$

$\left(\frac{2}{3}\right)^x = \frac{9}{4} \rightarrow \left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^{-2} \rightarrow x = -2$

$16\left(\frac{1}{4}\right)^x - 10\left(\frac{1}{2}\right)^x + 1 = 0$

$t = \left(\frac{1}{2}\right)^x$

$16t^2 - 10t + 1 = 0$

$\frac{1}{4}\Delta = 25 - 16 = 9$

$t_{1,2} = \frac{5 \pm 3}{16} = \begin{cases} \frac{1}{8} \\ \frac{1}{2} \end{cases}$

$\left(\frac{1}{2}\right)^x = \frac{1}{8} \rightarrow \left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^3 \rightarrow x = 3$

$\left(\frac{1}{2}\right)^x = \frac{1}{2} \rightarrow x = 1$

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$4^x - 2\cdot 2^x + 1 = 0$

$t = 2^x$

$t^2 - 2t + 1 = 0$

$(t-1)^2 = 0$

$t = 1 \rightarrow 2^x = 1 \rightarrow 2^x = 2^0 \rightarrow x = 0$

$4^x + 2^{x+1} - 24 = 0$

$t = 2^x$

$t^2 + 2t - 24 = 0$

$t_{1,2} = -1 \pm 5 = \begin{cases} -6 \\ +4 \end{cases}$

$t = -6 \rightarrow 2^x = -6$ impossibile

$t = 4 \rightarrow 2^x = 2^2 \rightarrow x = 2$

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$3\cdot 2^{2x-1} - 2^x = 4$

$t = 2^x$

$\frac{3}{2}t^2 - t - 4 = 0$

$3t^2 - 2t - 8 = 0$

$t_{1,2} = (1 \pm 5)/3 = \begin{cases} -\frac{4}{3} \\ 2 \end{cases}$

$2^x = -\frac{4}{3}$ impossibile

$2^x = 2 \rightarrow x = 1$

$9^x - 4\cdot 3^{x+1} + 27 = 0$

$3^{2x} - 12\cdot 3^x + 27 = 0 \qquad t = 3^x$

$t^2 - 12t + 27 = 0$

$t_{1,2} = 6 \pm 3 = \begin{cases} 3 \\ 9 \end{cases}$

$3^x = 3 \rightarrow x = 1$

$3^x = 9 \rightarrow 3^x = 3^2 \rightarrow x = 2$

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$2\cdot 3^x - 9^x = 1 \qquad t = 3^x$

$-t^2 + 2t - 1 = 0$

$(t-1)^2 = 0$

$t = 1 \rightarrow 3^x = 3^0 \rightarrow x = 0$

$(2^x + 4)(3^x - 9) = 0$

$2^x + 4 = 0 \rightarrow 2^x = -4$ impossibile

$3^x - 9 = 0 \rightarrow 3^x = 3^2 \rightarrow x = 2$

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$4^x + 2^{x+1} - 3 = 0 \qquad t = 2^x$

$t^2 + 2t - 3 = 0$

$t_{1,2} = -1 \pm 2 = \begin{cases} -3 \\ +1 \end{cases}$

$2^x = -3$ impossibile

$2^x = 1 \rightarrow x = 0$

$4^{x+2} - 5\cdot 2^{x+1} + 1 = 0 \qquad t = 2^x$

$16t^2 - 10t + 1 = 0$

$t_{1,2} = (5 \pm 3)/16 = \begin{cases} \frac{1}{8} \\ \frac{1}{2} \end{cases}$

$2^x = \frac{1}{8} \rightarrow 2^x = 2^{-3} \rightarrow x = -3$

$2^x = \frac{1}{2} \rightarrow 2^x = 2^{-1} \rightarrow x = -1$

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$\frac{3^{2-x} - 3^{1-x}}{9^{x+1} - 3^{2x+1}} = 27^{1+3x} \qquad t = 3^x$

$\frac{\frac{9}{t} - \frac{3}{t}}{9t^2 - 3t^2} = 3^3 \cdot t^9$

$\frac{6}{t} \cdot \frac{1}{6t^2} = 3^3 \cdot t^9$

$t^{12} = 3^{-3}$

$3^{12x} = 3^{-3} \rightarrow 12x = -3 \rightarrow x = -\frac{1}{4}$

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$$\frac{4^{x+1}-61}{4^{x-2}} = 3$$

$$4 \cdot 4^x - 61 = \frac{3}{16} \cdot 4^x$$

$$64 \cdot 4^x - 3 \cdot 4^x = 61 \cdot 4^2$$

$$61 \cdot 4^x = 61 \cdot 4^2$$

$$4^x = 4^2 \rightarrow x = 2$$

$$9^{x+1} = \frac{3^{x+2} - 3^{x+1}}{2}$$

$$9 \cdot 3^{2x} = \frac{9 \cdot 3^x - 3 \cdot 3^x}{2} \qquad 3^x = t$$

$$18t^2 = 6t$$

$$3t^2 - t = 0$$

$$t_1 = 0 \rightarrow 3^x = 0 \quad \text{impossibile}$$

$$t_2 = \frac{1}{3} \rightarrow 3^x = 3^{-1} \rightarrow x = -1$$

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$$3^{x+1} + \frac{3}{3^{1-x}} = \frac{6^{x+2} + 6^x}{37}$$

$$3 \cdot 3^x + 3 \cdot 3^{x-1} = \frac{36 \cdot 6^x + 6^x}{37}$$

$$3 \cdot 3^x + \cancel{3} \cdot 3^x \cdot \frac{1}{\cancel{3}} = \frac{\cancel{37} \cdot 6^x}{\cancel{37}}$$

$$4 \cdot 3^x = 6^x \rightarrow \frac{4 \cdot 3^x}{3^x} = \frac{6^x}{3^x} \rightarrow 4 = 2^x \rightarrow 2^x = 2^2 \rightarrow x = 2$$

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$$\frac{1}{4} \cdot 7^{2-x} = \frac{7}{21 + \sqrt{7^x}}$$

$$\left(21 + 7^{\frac{1}{2}x}\right) \cdot \cancel{49} \cdot 7^{-x} = 4 \cdot \cancel{7} \qquad t = 7^{\frac{x}{2}}$$

$$\left(21 + t\right) \frac{7}{t^2} = 4$$

$$147 + 7t = 4t^2$$

$$4t^2 - 7t - 147 = 0$$

$$\Delta = 2401 = 49^2$$

$$t_{1,2} = \frac{7 \pm 49}{8} = \begin{cases} -\frac{21}{4} \\ 7 \end{cases}$$

$$7^{\frac{x}{2}} = -\frac{21}{4} \quad \text{impossibile}$$

$$7^{\frac{x}{2}} = 7 \rightarrow x = 2$$

$$\frac{16 + 64^{\frac{1}{x}}}{2} = 12 - 8^{\frac{1}{x}} \qquad t = 8^{\frac{1}{x}} = 2^{\frac{3}{x}}$$

$$16 + t^2 = 24 - 2t$$

$$t^2 + 2t - 8 = 0$$

$$t_{1,2} = -1 \pm 3 = \begin{cases} -4 \\ 2 \end{cases}$$

$$2^{\frac{3}{x}} = -4 \quad \text{impossibile}$$

$$2^{\frac{3}{x}} = 2 \rightarrow \frac{3}{x} = 1 \rightarrow x = 3$$

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$$2^{1-x} + 2^{1+x} = 4 \qquad t = 2^x$$

$$\frac{2}{t} + 2t = 4$$

$$\frac{1}{t} + t = 2$$

$$1 + t^2 = 2t$$

$$t^2 - 2t + 1 = 0$$

$$(t-1)^2 = 0$$

$$t = 1$$

$$2^x = 2^0$$

$$x = 0$$

$$100^x - 6 \cdot 10^x = 5\left(10^x - 2\right) \qquad t = 10^x$$

$$t^2 - 6t = 5(t-2)$$

$$t^2 - 6t = 5t - 10$$

$$t^2 - 11t + 10 = 0$$

$$\Delta = 121 - 40 = 81 = 9^2$$

$$t_{1,2} = \frac{11 \pm 9}{2} = \begin{cases} 1 \\ 10 \end{cases}$$

$$10^x = 1 \rightarrow 10^x = 10^0 \rightarrow x = 0$$

$$10^x = 10 \rightarrow x = 1$$

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$$\frac{3^{2x}+2\cdot 3^x+1}{3^{x+2}-3^x}=\frac{2}{3}$$

$t=3^x$

$$\frac{t^2+2t+1}{9t-t}=\frac{2}{3}$$

$$3t^2+6t+3=16t$$

$$3t^2-10t+3=0$$

$$\frac{1}{4}\Delta=25-9=16$$

$$t_{1,2}=\frac{5\pm 4}{3}=\begin{cases}\frac{1}{3}\\ 3\end{cases}$$

$3^x=\frac{1}{3}\rightarrow x=-1$

$3^x=3\rightarrow x=1$

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$$2^{3-x}+2^{x+1}=17 \qquad t=2^x$$

$$\frac{8}{t}+2t=17$$

$$8+2t^2=17t$$

$$2t^2-17t+8=0$$

$$t_{1,2}=\frac{17\pm 15}{4}=\begin{cases}\frac{1}{2}\\ 8\end{cases}$$

$2^x=\frac{1}{2}\rightarrow x=-1$

$2^x=8\rightarrow x=3$

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$$\frac{1}{4^{1-3x}}+2^{3x+1}=\frac{1}{4^{2-3x}}+2^{3x+3} \qquad t=2^{3x}$$

$$\frac{1}{\frac{4}{t^2}}+2t=\frac{1}{\frac{16}{t^2}}+8t$$

$$\frac{t^2}{4}+2t=\frac{t^2}{16}+8t$$

$$\frac{3}{16}t^2-6t=0$$

$$t^2-32t=0$$

$$t_{1,2}=\begin{cases}0\\ 32\end{cases}$$

$2^{3x}=0$ impossibile

$2^{3x}=2^5\rightarrow 3x=5\rightarrow x=\frac{5}{3}$

$$\sqrt{3^x}-9=8\cdot\sqrt[4]{3^x} \qquad t=\sqrt[4]{3^x}=3^{\frac{x}{4}}$$

$$t^2-8t-9=0$$

$$t_{1,2}=4\pm 5=\begin{cases}-1\\ 9\end{cases}$$

$3^{\frac{x}{4}}=-1$ impossibile

$3^{\frac{x}{4}}=3^2\rightarrow x=8$

$$5^{\sqrt{x}}-\frac{1}{5}+5^{1-\sqrt{x}}=25 \qquad t=5^{\sqrt{x}}$$

$$t-\frac{1}{5}+\frac{5}{t}=25$$

$$5t^2-t+25=125t$$

$$5t^2-126t+25=0$$

$$\frac{\Delta}{4}=3844=62^2$$

$$t_{1,2}=\frac{63\pm 62}{5}=\begin{cases}\frac{1}{5}\\ 25\end{cases}$$

$5^{\sqrt{x}}=5^{-1}\rightarrow \sqrt{x}=-1$ impossibile

$5^{\sqrt{x}}=5^2\rightarrow \sqrt{x}=2\rightarrow x=4$

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$\frac{2^{2x}}{1+2^x} = 1 - \frac{2^x}{2^x+1}$ $\quad t = 2^x$

$\frac{t^2}{1+t} = 1 - \frac{t}{t+1}$

$t^2 = t+1-t$

$t^2 = 1$ $\quad t = \begin{cases} +1 \\ -1 \end{cases}$ $\quad 2^x = 1 \rightarrow x = 0$ $\quad 2^x = -1$ impossibile

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$\frac{1}{4^{2x}+4} - \frac{1}{8} = \frac{4^x - 2}{2^{4x+1}+8}$ $\quad t = 2^{2x}$

$\frac{1}{t^2+4} - \frac{1}{8} = \frac{t-2}{2t^2+8}$

$\frac{8-t^2-4}{8(t^2+4)} = \frac{4t-8}{8(t^2+4)}$

$4 - t^2 = 4t - 8$

$t^2 + 4t - 12 = 0$

$t_{1,2} = -2 \pm 4 = \begin{cases} -6 \\ 2 \end{cases}$ $\quad 2^{2x} = -6$ impossibile $\quad 2^{2x} = 2 \rightarrow x = \frac{1}{2}$

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$6^{x+1} + 6^{x-1} + 6^x = \frac{43}{6^{x-2}}$

$6^x \left(6 + \frac{1}{6} + 1\right) = \frac{43}{\frac{6^x}{36}}$

$6^x \cdot \frac{43}{6} = \frac{43 \cdot 36}{6^x}$

$6^{2x} = 6^3 \rightarrow 2x = 3 \rightarrow x = \frac{3}{2}$

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$4^{x-1} = 5 - 4^{2-x}$ $\quad t = 4^x$

$\frac{1}{4}t = 5 - \frac{16}{t}$

$t^2 = 20t - 64$

$t^2 - 20t + 64 = 0$

$t_{1,2} = 10 \pm 6 = \begin{cases} 4 \\ 16 \end{cases}$

$4^x = 4 \rightarrow x = 1$

$4^x = 16 \rightarrow x = 2$

$2\left(5^x - \frac{5^x - 1}{5^x + 1}\right) = 3 \cdot 5^x - 1$ $\quad t = 5^x$

$2\left(t - \frac{t-1}{t+1}\right) = 3t - 1$

$2\,\frac{t^2+t-t+1}{t+1} = \frac{3t^2 - t + 3t - 1}{t+1}$

$2t^2 + 2 = 3t^2 + 2t - 1$

$t^2 + 2t - 3 = 0$

$t_{1,2} = -1 \pm 2 = \begin{cases} -3 \\ 1 \end{cases}$ $\quad 5^x = -3$ impossibile $\quad 5^x = 1 \rightarrow x = 0$

$\frac{8 - 9^x}{1 + 3^{3-2x}} + \frac{1}{4} = 0$ $\quad t = 3^{2x}$

$\frac{8-t}{1+27 \cdot \frac{1}{t}} + \frac{1}{4} = 0$

$\frac{8t - t^2}{t+27} + \frac{1}{4} = 0$

$32t - 4t^2 + t + 27 = 0$

$4t^2 - 33t - 27 = 0$

$\Delta = 1521 = 39^2$

$t_{1,2} = \frac{33 \pm 39}{8} = \begin{cases} -\frac{3}{4} \\ 9 \end{cases}$

$3^{2x} = -\frac{3}{4}$ impossibile

$3^{2x} = 3^2 \rightarrow x = 1$

$\left(4^{x-1}\right)^{x+1} = 8^{x^2-2}$

$4^{x^2-1} = 8^{x^2-2}$

$2^{2x^2-2} = 2^{3x^2-6}$

$2x^2 - 2 = 3x^2 - 6$

$-x^2 + 4 = 0$

$x = \pm 2$

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$2^x + 3^x = \frac{1}{3}2^x + \frac{1}{2}3^x$

$2^x\left(1-\frac{1}{3}\right) = 3^x\left(\frac{1}{2}-1\right)$

$2^x \cdot \frac{2}{3} = 3^x\left(-\frac{1}{2}\right)$

$\left(\frac{2}{3}\right)^x = -\frac{3}{4}$ impossibile

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$6^x - 3^x - 3\cdot 2^x + 3 = 0$

$3^x\cdot 2^x - 3^x - 3\cdot 2^x + 3 = 0$

$3^x(2^x-1) - 3(2^x-1) = 0$

$(3^x-3)(2^x-1) = 0$ $\quad 3^x = 3 \rightarrow x = 1$ $\quad 2^x = 1 \rightarrow x = 0$

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$2\cdot 2^{3x} - 3\cdot 2^{2x} - 3\cdot 2^x + 2 = 0$

$t = 2^x$

$2t^3 - 3t^2 - 3t + 2 = 0$

	2	-3	-3	2
-1		-2	5	-2
	2	-5	2	0

$(t+1)(2t^2-5t+2) = 0$

$t_1 = -1$

$t_{2,3} = \frac{5\pm 3}{4} = \begin{cases} \frac{1}{2} \\ 2 \end{cases}$

$2^x = -1$ impossibile

$2^x = \frac{1}{2} \rightarrow x = -1$

$2^x = 2 \rightarrow x = 1$

$27^x - 9^x - 3^{x+2} + 9 = 0$ $\quad t = 3^x$

$t^3 - t^2 - 9t + 9 = 0$

$t^2(t-1) - 9(t-1) = 0$

$(t^2-9)(t-1) = 0$

$t_1 = 1$ $\quad t_{2,3} = \begin{cases} -3 \\ +3 \end{cases}$

$3^x = 1 \rightarrow x = 0$

$3^x = -3$ impossibile

$3^x = 3 \rightarrow x = 1$

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$\left(\frac{1}{8}\right)^x - \left(\frac{1}{4}\right)^x - 4\left(\frac{1}{2}\right)^x + 4 = 0$ $\quad t = \left(\frac{1}{2}\right)^x$

$t^3 - t^2 - 4t + 4 = 0$

$t^2(t-1) - 4(t-1) = 0$

$(t^2-4)(t-1) = 0$ $\quad t_1 = 1$ $\quad t_{2,3} = \begin{cases} -2 \\ +2 \end{cases}$

$\left(\frac{1}{2}\right)^x = 1 \rightarrow x = 0$

$\left(\frac{1}{2}\right)^x = -2$ impossibile

$\left(\frac{1}{2}\right)^x = 2 \rightarrow 2^{-x} = 2 \rightarrow x = -1$

$2^{3x+1} - 7\cdot 4^x + 7\cdot 2^x - 2 = 0$ $\quad t = 2^x$

$2t^3 - 7t^2 + 7t - 2 = 0$

	2	-7	7	-2
+1		+2	-5	2
	2	-5	2	0

$(t-1)(2t^2-5t+2) = 0$

$t_1 = 1$ $\quad t_{2,3} = \frac{5\pm 3}{4} = \begin{cases} \frac{1}{2} \\ 2 \end{cases}$

$2^x = 1 \rightarrow x = 0$

$2^x = \frac{1}{2} \rightarrow x = -1$

$2^x = 2 \rightarrow x = 1$

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$\left(\frac{5}{2}\right)^{x+1} + \left(\frac{2}{5}\right)^{x-1} = \frac{29}{4}$ $\quad t = \left(\frac{5}{2}\right)^x$

$\frac{5}{2}t + \frac{5}{2}\frac{1}{t} = \frac{29}{4}$

$10t^2 + 10 = 29t$

$10t^2 - 29t + 10 = 0$

$t_{1,2} = \frac{29 \pm 21}{20} = \begin{cases} \frac{8}{20} = \frac{2}{5} \\ \frac{50}{20} = \frac{5}{2} \end{cases}$

$\left(\frac{5}{2}\right)^x = \frac{2}{5} \rightarrow x = -1$

$\left(\frac{5}{2}\right)^x = \frac{5}{2} \rightarrow x = 1$

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$81^{x-1} \cdot \sqrt[3]{9^{2-x}} = \frac{\left(3\sqrt[4]{3^{2x-1}}\right)^2}{\sqrt{27^{x+1}}}$

$3^{4x-4} \cdot 3^{\frac{4-2x}{3}} = \frac{3^2 \cdot 3^{\frac{2x-1}{2}}}{3^{\frac{3x+3}{2}}}$

$4x - 4 + \frac{4-2x}{3} = 2 + \frac{2x-1}{2} - \frac{3x+3}{2}$

$\frac{10x-8}{3} = \frac{-x}{2}$

$20x - 16 = -3x$

$23x = 16 \rightarrow x = \frac{16}{23}$

$\frac{2^{2x-1}+3}{2^x+1} = 2^x - \frac{1}{3}$ $\quad t = 2^x$

$\frac{\frac{1}{2}t^2+3}{t+1} = t - \frac{1}{3}$

$\frac{1}{2}t^2 + 3 = t^2 + \frac{2}{3}t - \frac{1}{3}$

$3t^2 + 18 = 6t^2 + 4t - 2$

$3t^2 + 4t - 20 = 0$

$t_{1,2} = \frac{-2 \pm 8}{3} = \begin{cases} -\frac{10}{3} \\ 2 \end{cases}$

$2^x = -\frac{10}{3}$ impossibile

$2^x = 2 \rightarrow x = 1$

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$4^{x-1} = 5 - 4^{2-x}$ $\quad t = 4^x$

$\frac{1}{4}t = 5 - \frac{16}{t}$

$t^2 = 20t - 64$

$t^2 - 20t + 64 = 0$

$t_{1,2} = 10 \pm 6 = \begin{cases} 4 \\ 16 \end{cases}$

$4^x = 4 \rightarrow x = 1$

$4^x = 16 \rightarrow x = 2$

$\left(2^x - 16^{2-3x}\right)^2 + \left(169x^2 - 64\right)^8 = 0$

$\begin{cases} 169x^2 - 64 = 0 \\ 2^x - 16^{2-3x} = 0 \end{cases} \quad \begin{cases} x = \pm\frac{8}{13} \\ x = \frac{8}{13} \end{cases} \rightarrow x = \frac{8}{13}$

$2^x - 2^{8-12x} = 0$

$2^{13x} = 2^8 \rightarrow 13x = 8 \rightarrow x = \frac{8}{13}$

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$\left(2^{x+1} + 2\right)\left(9^x - 3\right) = 0$

$2^{x+1} = -2$ impossibile

$9^x = 3 \rightarrow 3^{2x} = 3 \rightarrow 2x = 1 \rightarrow x = \frac{1}{2}$

$\left(4^x - 8\right)\left(3^x + 81\right)\left(5^x - \frac{1}{25}\right) = 0$

$4^x - 8 = 0 \rightarrow 2^{2x} = 2^3 \rightarrow x = \frac{3}{2}$

$3^x + 81 = 0 \rightarrow 3^x = -81$ impossibile

$5^x - \frac{1}{25} = 0 \rightarrow 5^x = 5^{-2} \rightarrow x = -2$

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$9 \cdot 3^{2x} = 5^{x+1}$

$9 \cdot 9^x = 5 \cdot 5^x$

$\frac{9^x}{5^x} = \frac{5}{9}$

$\left(\frac{9}{5}\right)^x = \left(\frac{9}{5}\right)^{-1}$

$x = -1$

$3^x \cdot 5^{x-2} = 9$

$3^x \cdot 5^x \cdot \frac{1}{5^2} = 3^2$

$15^x = 15^2$

$x = 2$

$2^{2x+4} \cdot 3^x = \frac{2}{3^{x+3}}$

$2^4 \cdot 4^x \cdot 3^x \cdot 3^{x+3} = 2$

$2^4 \cdot 3^3 \cdot 4^x \cdot 9^x = 2$

$36^x = \frac{1}{2^3 \cdot 3^3}$

$6^{2x} = 6^{-3}$

$2x = -3 \rightarrow x = -\frac{3}{2}$

$3^x = \frac{5^{x+1}}{3}$

$3^x = \frac{5 \cdot 5^x}{3}$

$\frac{3^x}{5^x} = \frac{5}{3}$

$\left(\frac{3}{5}\right)^x = \left(\frac{3}{5}\right)^{-1} \rightarrow x = -1$