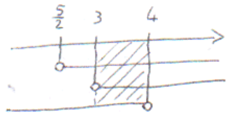
n. 44 p. 484

$$\log_3 \log_3 (2x-5) < 0$$

$$\begin{cases} \log_3(2x-5) > 0 \\ \log_3 \log_3 (2x-5) < \log_3 1 \end{cases} \rightarrow \begin{cases} 2x-5>0 \\ \log_3(2x-5) > \log_3 1 \\ \log_3(2x-5) < 1 \end{cases} \rightarrow \begin{cases} x > \frac{5}{2} \\ 2x-5>1 \\ \log_3(2x-5) < \log_3 3 \end{cases} \rightarrow \begin{cases} x>\frac{5}{2} \\ 2x>6 \\ 2x-5<3 \end{cases} \rightarrow$$

$$\rightarrow \begin{cases} x>\frac{5}{2} \\ x>3 \\ 2x<8 \end{cases} \rightarrow \begin{cases} x>\frac{5}{2} \\ x>3 \\ x<4 \end{cases}$$



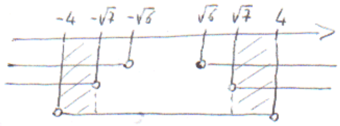
$S: \quad 3<x<4$

n. 45 p. 484

$$\log \log (x^2-6) < 0$$

$$\begin{cases} \log(x^2-6) > 0 \\ \log \log (x^2-6) < \log 1 \end{cases} \rightarrow \begin{cases} x^2-6>0 \\ \log(x^2-6) > \log 1 \\ \log(x^2-6) < 1 \end{cases} \rightarrow \begin{cases} x<-\sqrt{6} \ \vee \ x>\sqrt{6} \\ x^2-6>1 \\ \log(x^2-6) < \log 10 \end{cases} \rightarrow$$

$$\rightarrow \begin{cases} x<-\sqrt{6} \ \vee \ x>\sqrt{6} \\ x^2-7>0 \\ x^2-6<10 \end{cases} \rightarrow \begin{cases} x<-\sqrt{6} \ \vee \ x>\sqrt{6} \\ x<-\sqrt{7} \ \vee \ x>\sqrt{7} \\ -4<x<4 \end{cases}$$



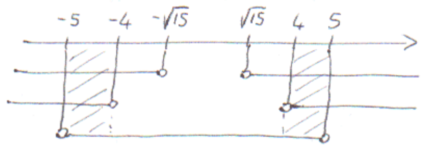
$S: \quad -4<x<-\sqrt{7} \quad \vee \quad \sqrt{7}<x<4$

n. 46 p. 484

$$\log \log (x^2-15) < 0$$

$$\begin{cases} \log(x^2-15) > 0 \\ \log \log (x^2-15) < \log 1 \end{cases} \rightarrow \begin{cases} x^2-15>0 \\ \log(x^2-15) > \log 1 \\ \log(x^2-15) < 1 \end{cases} \rightarrow \begin{cases} x<-\sqrt{15} \ \vee \ x>\sqrt{15} \\ x^2-15>1 \\ \log(x^2-15) < \log 10 \end{cases} \rightarrow$$

$$\rightarrow \begin{cases} x<-\sqrt{15} \ \vee \ x>\sqrt{15} \\ x^2-16>0 \\ x^2-15<10 \end{cases} \rightarrow \begin{cases} x<-\sqrt{15} \ \vee \ x>\sqrt{15} \\ x<-4 \ \vee \ x>4 \\ x<-5 \ \vee \ x>5 \end{cases}$$



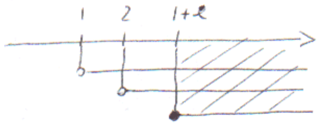
$S: \quad -5<x<-4 \quad \vee \quad 4<x<5$

n. 47 p. 484

$\log \log (x-1) \geq 0$

$$\begin{cases} \log(x-1) > 0 \\ \log\log(x-1) \geq \log 1 \end{cases} \rightarrow \begin{cases} x-1>0 \\ \log(x-1) > \log 1 \\ \log(x-1) \geq 1 \end{cases} \rightarrow \begin{cases} x>1 \\ x-1>1 \\ \log(x-1) \geq \log e \end{cases} \rightarrow$$

$$\rightarrow \begin{cases} x>1 \\ x>2 \\ x-1 \geq e \end{cases} \rightarrow \begin{cases} x>1 \\ x>2 \\ x \geq 1+e \end{cases}$$



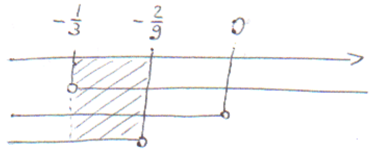
S: $x \geq 1+e$

n. 48 p. 484

$\log_3 \log_{\frac{1}{3}} (1+3x) > 0$

$$\begin{cases} \log_{\frac{1}{3}}(1+3x) > 0 \\ \log_3 \log_{\frac{1}{3}}(1+3x) > \log_3 1 \end{cases} \rightarrow \begin{cases} 1+3x>0 \\ \log_{\frac{1}{3}}(1+3x) > \log_{\frac{1}{3}} 1 \\ \log_{\frac{1}{3}}(1+3x) > 1 \end{cases} \rightarrow \begin{cases} x > -\frac{1}{3} \\ 1+3x<1 \\ \log_{\frac{1}{3}}(1+3x) > \log_{\frac{1}{3}} \frac{1}{3} \end{cases} \rightarrow$$

$$\rightarrow \begin{cases} x>-\frac{1}{3} \\ 3x<0 \\ 1+3x<\frac{1}{3} \end{cases} \rightarrow \begin{cases} x>-\frac{1}{3} \\ x<0 \\ 3+9x<1 \end{cases} \rightarrow \begin{cases} x>-\frac{1}{3} \\ x<0 \\ x<-\frac{2}{9} \end{cases}$$



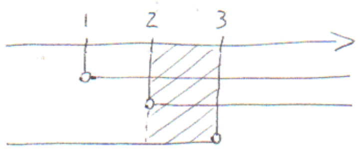
S: $-\frac{1}{3} < x < -\frac{2}{9}$

n. 49 p. 484

$\log_3 \log_2 (x-1) < 0$

$$\begin{cases} \log_2(x-1) > 0 \\ \log_3 \log_2(x-1) < 0 \end{cases} \rightarrow \begin{cases} x-1>0 \\ \log_2(x-1) > \log_2 1 \\ \log_3\log_2(x-1) < \log_3 1 \end{cases} \rightarrow \begin{cases} x>1 \\ x-1>1 \\ \log_2(x-1)<1 \end{cases} \rightarrow$$

$$\rightarrow \begin{cases} x>1 \\ x>2 \\ \log_2(x-1) < \log_2 2 \end{cases} \rightarrow \begin{cases} x>1 \\ x>2 \\ x-1<2 \end{cases} \rightarrow \begin{cases} x>1 \\ x>2 \\ x<3 \end{cases}$$



S: $2 < x < 3$

n. 50 p. 485

$$\log_{\frac{1}{2}}(x^2+2)+\log_2(x-2)\le -2\log_4(x+1)$$

C.A.

$$\begin{cases} x^2+2>0 \\ x-2>0 \\ x+1>0 \end{cases} \to \begin{cases} \forall x\in\mathbb{R} \\ x>2 \\ x>-1 \end{cases} \to x>2$$

$$\frac{\log_2(x^2+2)}{\log_2\frac{1}{2}}+\log_2(x-2)\le -2\,\frac{\log_2(x+1)}{\log_2 4}$$

$$-\log_2(x^2+2)+\log_2(x-2)\le -\log_2(x+1)$$

$$\log_2(x-2)+\log_2(x+1)\le \log_2(x^2+2)$$

$$\log_2(x^2-x-2)\le \log_2(x^2+2)$$

$$x^2-x-2\le x^2+2$$

$$x\ge -4 \longrightarrow \begin{cases} x>2 \\ x\ge -4\end{cases} \to \boxed{x>2}$$

n. 51 pag. 485

$$\log(4^{1-x}+2)-\log(2^{2x+1}-3)<\log 2$$

$$\begin{cases} 2^{2x+1}-3>0 \\ \log(4^{1-x}+2)<\log 2+\log(2\cdot 4^x-3)\end{cases} \to \begin{cases} 2\cdot 4^x>3 \\ \log(4^{1-x}+2)<\log(4\cdot 4^x-6)\end{cases} \to \begin{cases} 4^x>\frac{3}{2} \\ \frac{4}{4^x}+2<4\cdot 4^x-6\end{cases} \to$$

$$\to \begin{cases} x\log 4>\log\frac{3}{2} \\ 4+8\cdot 4^x-4\cdot 4^{2x}<0\end{cases} \to \begin{cases} x>\frac{\log 3-\log 2}{\log 4} \\ 4^{2x}-2\cdot 4^x-1>0\end{cases} \to \begin{cases} x>\frac{\log 3-\log 2}{\log 4}\approx 0{,}29248.. \\ x>\frac{\log(1+\sqrt{2})}{\log 4}\approx 0{,}63577..\end{cases} \to \boxed{x>\frac{\log(1+\sqrt{2})}{\log 4}}$$

2° diseq. $t=4^x$

$$t^2-2t-1>0 \to t_{1,2}=1\pm\sqrt{2} \to t<1-\sqrt{2} \lor t>1+\sqrt{2}$$

$$4^x<1-\sqrt{2} \lor 4^x>1+\sqrt{2}$$

impossibile $\quad x>\frac{\log(1+\sqrt{2})}{\log 4}$

n. 52 pag. 485

$$x^{\log\sqrt{x}}>100$$

$$\begin{cases} x>0 \\ \log x^{\log\sqrt{x}}>\log 100\end{cases}$$

$$\begin{cases} x>0 \\ \log\sqrt{x}\,\log x>2\end{cases}$$

$$\begin{cases} x>0 \\ \frac{1}{2}\log^2 x>2\end{cases}$$

$$\begin{cases} x>0 \\ \log^2 x>4\end{cases}$$

$$\begin{cases} x>0 \\ \log x<-2 \lor \log x>2\end{cases}$$

$$\begin{cases} x>0 \\ x<\frac{1}{100} \lor x>100\end{cases}$$

$$\boxed{0<x<\frac{1}{100} \lor x>100}$$

$$\log_3(2-3^x)+x>0$$

$$\begin{cases} 2-3^x>0 \\ \log_3(2-3^x)>-x\log_3 3\end{cases}$$

$$\begin{cases} 3^x<2 \\ 2-3^x>3^{-x}\end{cases}$$

$$\begin{cases} x\log 3<\log 2 \\ 2\cdot 3^x-3^{2x}>1\end{cases}$$

$$\begin{cases} x<\frac{\log 2}{\log 3} \\ 3^{2x}-2\cdot 3^x+1<0\end{cases}$$

$$\begin{cases} x<\frac{\log 2}{\log 3} \\ (3^x-1)^2<0\end{cases} \to \begin{cases} x<\frac{\log 2}{\log 3} \\ \emptyset\end{cases} \to \boxed{S=\emptyset}$$

n. 53 pag. 485

$$|1-\log_3 x| < 1+\log_3 x$$

$t=\log_3 x$

$$|1-t| < 1+t$$

$$\begin{cases}1-t\geq 0\\ 1-t<1+t\end{cases} \vee \begin{cases}1-t<0\\ t-1<1+t\end{cases}$$

$$\begin{cases}t\leq 1\\ t>0\end{cases} \vee \begin{cases}t>1\\ \forall t\in\mathbb{R}\end{cases}$$

$$0<t\leq 1 \quad \vee \quad t>1$$

$$t>0$$

$$\log_3 x>0$$

$$\begin{cases}x>0\\ \log_3 x>\log_3 1\end{cases}$$

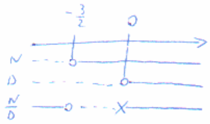
$$\begin{cases}x>0\\ x>1\end{cases} \rightarrow \boxed{x>1}$$

$$\frac{|\log x+1|-2}{\log x}<1$$

$t=\log x \qquad \frac{|t+1|-2}{t}-1<0$

$$\begin{cases}t+1\geq 0\\ \frac{t+1-2-t}{t}<0\end{cases} \vee \begin{cases}t+1<0\\ \frac{-t-1-2-t}{t}<0\end{cases}$$

$$\begin{cases}t\geq -1\\ \frac{-1}{t}<0\end{cases} \vee \begin{cases}t<-1\\ \frac{2t+3}{t}>0\end{cases}$$



$$\begin{cases}t\geq -1\\ t>0\end{cases} \vee \begin{cases}t<-1\\ t<-\frac{3}{2} \vee t>0\end{cases}$$

$$t>0 \quad \vee \quad t<-\frac{3}{2}$$

$$\log x<-\frac{3}{2} \quad \vee \quad \log x>0$$

$$\begin{cases}x>0\\ x<10^{-\frac{3}{2}} \vee x>1\end{cases} \rightarrow \boxed{0<x<10^{-\frac{3}{2}} \vee x>1}$$

n. 54 pag. 485

$$0<\log_2(5x+3)<1$$

$$\begin{cases}5x+3>0\\ \log_2 1<\log_2(5x+3)<\log_2 2\end{cases}$$

$$\begin{cases}x>-\frac{3}{5}\\ 1<5x+3<2\end{cases}$$

$$\begin{cases}x>-\frac{3}{5}\\ -2<5x<-1\end{cases}$$

$$\begin{cases}x>-\frac{3}{5}\\ -\frac{2}{5}<x<-\frac{1}{5}\end{cases}$$

$$\boxed{-\frac{2}{5}<x<-\frac{1}{5}}$$

$$-1<\log_3(x-1)<1$$

$$\begin{cases}x-1>0\\ \log_3\frac{1}{3}<\log_3(x-1)<\log_3 3\end{cases}$$

$$\begin{cases}x>1\\ \frac{1}{3}<x-1<3\end{cases}$$

$$\begin{cases}x>1\\ \frac{4}{3}<x<4\end{cases}$$

$$\boxed{\frac{4}{3}<x<4}$$

n. 55 pag. 485

$$\begin{cases} \log_{\frac{1}{2}} \log_2 (4-3x) \geq 0 \\ \log_2 \log_{\frac{1}{2}} (1-x) \geq 0 \end{cases}$$

$$\begin{cases} \log_2 (4-3x) > 0 \\ \log_2 (4-3x) \leq 1 \\ \log_{\frac{1}{2}} (1-x) > 0 \\ \log_{\frac{1}{2}} (1-x) > 1 \end{cases}$$

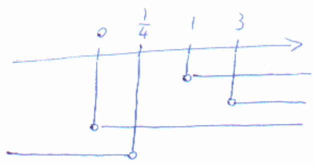
$$\begin{cases} 4-3x > 0 \\ 4-3x > 1 \\ 4-3x \leq 2 \\ 1-x > 0 \\ 1-x < 1 \\ 1-x < \frac{1}{2} \end{cases}$$

$$\begin{cases} x < \frac{4}{3} \\ x < 1 \\ x \geq \frac{2}{3} \\ x < 1 \\ x > 0 \\ x > \frac{1}{2} \end{cases} \qquad \boxed{\frac{2}{3} \leq x < 1}$$

$$\begin{cases} \log_2 (x-1) > 1 \\ \log_{\frac{1}{2}} x > 2 \end{cases}$$

$$\begin{cases} x-1 > 0 \\ x-1 > 2 \\ x > 0 \\ x < \frac{1}{4} \end{cases}$$

$$\begin{cases} x > 1 \\ x > 3 \\ x > 0 \\ x < \frac{1}{4} \end{cases}$$

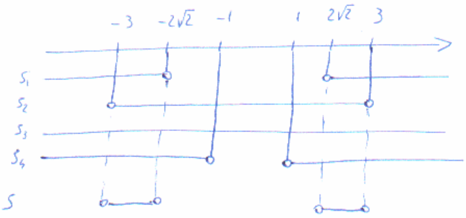


$$\boxed{S = \emptyset}$$

n. 56 pag. 485

$$\begin{cases} \log_{\frac{1}{2}} (x^2-8) > 0 \\ \log_3 (2+x^2) > 1 \end{cases}$$

$$\begin{cases} x^2-8 > 0 \\ x^2-8 < 1 \\ 2+x^2 > 0 \\ 2+x^2 > 3 \end{cases} \rightarrow \begin{cases} x < -2\sqrt{2} \quad \vee \quad x > 2\sqrt{2} \\ -3 < x < 3 \\ \forall x \in \mathbb{R} \\ x < -1 \quad \vee \quad x > 1 \end{cases}$$



$$\boxed{-3 < x < -2\sqrt{2} \quad \vee \quad 2\sqrt{2} < x < 3}$$

n. 57 pag. 485 (A)

$$\begin{cases} \dfrac{\log_2(x+2)}{\log_{\frac{1}{3}}(x^2-1)} \leq 0 \\ \log_2^2 x - 3\log_2 x + 2 > 0 \end{cases}$$

1ª diseq. $\dfrac{\log_2(x+2)}{\log_{\frac{1}{3}}(x^2-1)} \leq 0$

$N > 0$ C.A. $x > -2$

$\log_2(x+2) > 0$

$$\begin{cases} x+2 > 0 \\ x+2 > 1 \end{cases}$$

$$\begin{cases} x > -2 \\ x > -1 \end{cases} \rightarrow x > -1$$

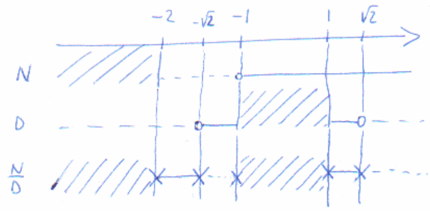
$D > 0$ C.A. $x < -1 \vee x > 1$

$\log_{\frac{1}{3}}(x^2-1) > 0$

$$\begin{cases} x^2-1 > 0 \\ x^2-1 < 1 \end{cases}$$

$$\begin{cases} x < -1 \vee x > 1 \\ -\sqrt{2} < x < \sqrt{2} \end{cases}$$

$-\sqrt{2} < x < -1 \vee 1 < x < \sqrt{2}$



$-\sqrt{2} < x < -1 \vee x > \sqrt{2}$

2ª diseq. $\log_2^2 x - 3\log_2 x + 2 > 0$

$t = \log_2 x$

$t^2 - 3t + 2 > 0$

$t_{1,2} = \dfrac{3 \pm 1}{2} = \begin{cases} 1 \\ 2 \end{cases}$

$t < 1 \vee t > 2$

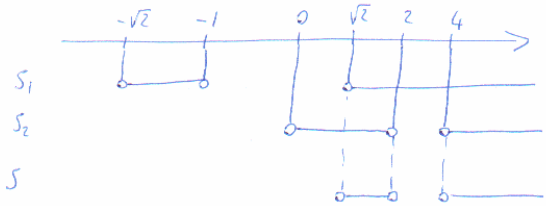
$\log_2 x < 1 \vee \log_2 x > 2$

$$\begin{cases} x > 0 \\ \log_2 x < \log_2 2 \end{cases} \vee \begin{cases} x > 0 \\ \log_2 x > \log_2 4 \end{cases}$$

$$\begin{cases} x > 0 \\ x < 2 \end{cases} \vee \begin{cases} x > 0 \\ x > 4 \end{cases}$$

$0 < x < 2 \vee x > 4$

Soluzioni del sistema:



$$\boxed{\sqrt{2} < x < 2 \vee x > 4}$$

n. 57 pag. 485 (B)

$$\begin{cases} \dfrac{2-\log_{\frac{1}{2}} x}{3+\log_2 x} \leq 0 \\ \log_{16} \dfrac{x+1}{2x-1} > \dfrac{1}{2} \end{cases}$$

1° diseq.

$$\frac{2+\log_2 x}{3+\log_2 x} \leq 0$$

C.A. $x>0$

$N>0$

$2+\log_2 x > 0$

$\log_2 x > \log_2 2^{-2}$

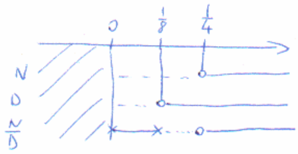
$x > \frac{1}{4}$

$D>0$

$3+\log_2 x > 0$

$\log_2 x > \log_2 2^{-3}$

$x > \frac{1}{8}$



$\frac{1}{8} < x \leq \frac{1}{4}$

2° diseq.

$$\log_{16} \frac{x+1}{2x-1} > \frac{1}{2}$$

C.A. $\frac{x+1}{2x-1} > 0$ $x<-1 \vee x>\frac{1}{2}$

$\log_{16} \frac{x+1}{2x-1} > \log_{16} 4$

$$\begin{cases} x<-1 \vee x>\frac{1}{2} \\ \frac{x+1}{2x-1} - 4 > 0 \end{cases}$$

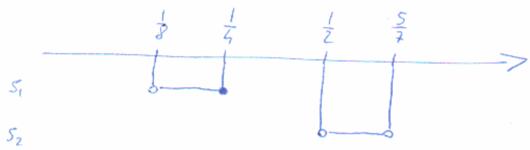
$$\begin{cases} x<-1 \vee x>\frac{1}{2} \\ \frac{-7x+5}{2x-1} > 0 \end{cases}$$

$$\begin{cases} x<-1 \vee x>\frac{1}{2} \\ \frac{7x-5}{2x-1} < 0 \end{cases}$$

$$\begin{cases} x<-1 \vee x>\frac{1}{2} \\ \frac{1}{2} < x < \frac{5}{7} \end{cases}$$

$\frac{1}{2} < x < \frac{5}{7}$

Soluzioni del sistema:



$$\boxed{S = \emptyset}$$

n. 58 pag. 485

$$\begin{cases} \log x - \dfrac{2}{\log x} + 1 \geq 0 \\ \dfrac{|\log x + 1| - 2}{\log x} < 1 \end{cases}$$

1ª diseq.

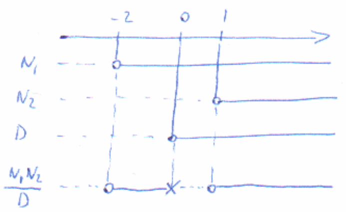
C.A. $x > 0 \quad x \neq 1$

$t = \log x$

$$t - \frac{2}{t} + 1 \geq 0$$

$$\frac{t^2 - 2 + t}{t} \geq 0$$

$$\frac{(t+2)(t-1)}{t} \geq 0$$



$$-2 \leq t < 0 \quad \vee \quad t \geq 1$$

$$\log 10^{-2} \leq \log x < \log 1 \quad \vee \quad \log x \geq \log 10$$

$$10^{-2} \leq x < 1 \quad \vee \quad x \geq 10$$

2ª diseq.

C.A. $x > 0 \quad x \neq 1$

$t = \log x$

$$\frac{|t+1| - 2}{t} - 1 < 0$$

$$\begin{cases} t + 1 \geq 0 \\ \dfrac{t + 1 - 2 - t}{t} < 0 \end{cases} \quad \vee \quad \begin{cases} t + 1 < 0 \\ \dfrac{-t - 1 - 2 - t}{t} < 0 \end{cases}$$

$$\begin{cases} t \geq -1 \\ \dfrac{-1}{t} < 0 \end{cases} \quad \vee \quad \begin{cases} t < -1 \\ \dfrac{2t + 3}{t} > 0 \end{cases}$$

$$\begin{cases} t \geq -1 \\ t > 0 \end{cases} \quad \vee \quad \begin{cases} t < -1 \\ t < -\frac{3}{2} \ \vee \ t > 0 \end{cases}$$

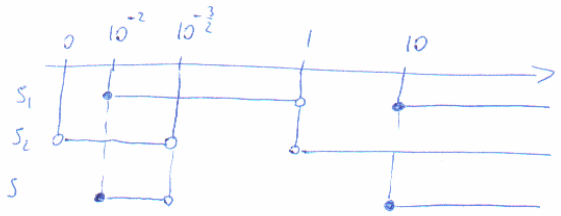
$$t > 0 \quad \vee \quad t < -\frac{3}{2}$$

$$\log x < -\frac{3}{2} \quad \vee \quad \log x > 0$$

$$\begin{cases} x > 0 \\ x < 10^{-\frac{3}{2}} \quad \vee \quad x > 1 \end{cases}$$

$$0 < x < 10^{-\frac{3}{2}} \quad \vee \quad x > 1$$

Soluzioni del sistema



$$\boxed{10^{-2} \leq x < 10^{-\frac{3}{2}} \quad \vee \quad x \geq 10}$$

n. 59 pag. 485

$$\frac{\log_3 x+1}{\log_3 x-1}-\frac{\log_3 x+2}{\log_3 x-2}+3\leq 0$$

$$t=\log_3 x$$

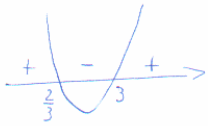
$$\frac{t+1}{t-1}-\frac{t+2}{t-2}+3\leq 0$$

$$\frac{\cancel{t^2}-2t+t-2-\cancel{t^2}-2t+t+2+3t^2-6t-3t+6}{(t-1)(t-2)}\leq 0$$

$$\frac{3t^2-11t+6}{(t-1)(t-2)}\leq 0$$

$N>0$

$3t^2-11t+6>0$ $\qquad t_{1,2}=\dfrac{11\pm 7}{6}=\begin{cases} -\ \dfrac{2}{3} \\ +\ 3\end{cases}$

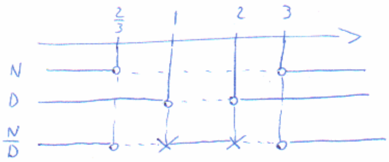


$t<\frac{2}{3} \quad \vee \quad t>3$

$D>0$

$(t-1)(t-2)>0$

$t<1 \quad \vee \quad t>2$



$\frac{2}{3}\leq t<1 \qquad \vee \qquad 2<t\leq 3$

$\frac{2}{3}\leq \log_3 x<1 \qquad \vee \qquad 2<\log_3 x\leq 3 \qquad\qquad$ C.A. $\quad x>0$

$\log_3 3^{\frac{2}{3}}\leq \log_3 x<\log_3 3 \qquad \vee \qquad \log_3 9<\log_3 x\leq \log_3 27$

$$\boxed{\sqrt[3]{9}\leq x<3 \qquad \vee \qquad 9<x\leq 27}$$

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$$\frac{\log_3 x+1}{\log_3 x-1}-\frac{\log_3 x+2}{\log_3 x-2}+3\le 0$$

$t=\log_3 x$

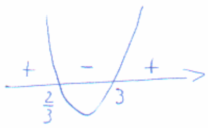
$$\frac{t+1}{t-1}-\frac{t+2}{t-2}+3\le 0$$

$$\frac{\cancel{t^2}-2t+t-2-\cancel{t^2}-2t+t+2+3t^2-6t-3t+6}{(t-1)(t-2)}\le 0$$

$$\frac{3t^2-11t+6}{(t-1)(t-2)}\le 0$$

$N>0$

$3t^2-11t+6>0$ $\qquad t_{1,2}=\frac{11\pm 7}{6}=\begin{cases} -\ \frac{2}{3} \\ +\ 3\end{cases}$

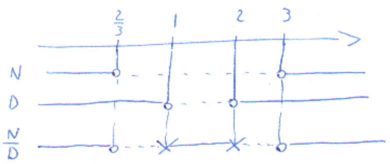


$t<\frac{2}{3} \quad \vee \quad t>3$

$D>0$

$(t-1)(t-2)>0$

$t<1 \quad \vee \quad t>2$



$\frac{2}{3}\le t<1 \qquad \vee \qquad 2<t\le 3$

$\frac{2}{3}\le \log_3 x<1 \qquad \vee \qquad 2<\log_3 x\le 3 \qquad\qquad$ C.A. $x>0$

$\log_3 3^{\frac{2}{3}}\le \log_3 x<\log_3 3 \qquad \vee \qquad \log_3 9<\log_3 x\le \log_3 27$

$$\boxed{\sqrt[3]{9}\le x<3 \qquad \vee \qquad 9<x\le 27}$$

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$$\frac{\log_2 x - 1}{\log_2 x + 1} - \frac{\log_2 x - 2}{\log_2 x + 2} - \frac{1}{3} \geq 0$$

$$t = \log_2 x$$

$$\frac{t-1}{t+1} - \frac{t-2}{t+2} - \frac{1}{3} \geq 0$$

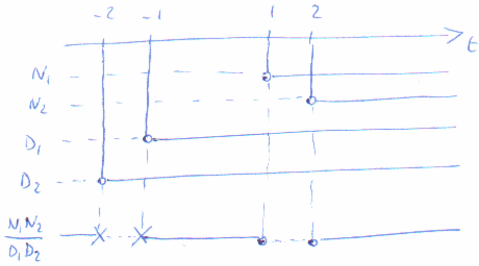
$$\frac{3(t^2+t-2) - 3(t^2-t-2) - (t^2+3t+2)}{\cancel{3}(t+1)(t+2)} \geq 0$$

$$\frac{\cancel{3t^2}+3t\cancel{-6}\cancel{-3t^2}\cancel{+3t}\cancel{+6}-t^2\cancel{-3t}-2}{(t+1)(t+2)} \geq 0$$

$$\frac{-t^2+3t-2}{(t+1)(t+2)} \geq 0$$

$$\frac{t^2-3t+2}{(t+1)(t+2)} \leq 0$$

$$\frac{(t-1)(t-2)}{(t+1)(t+2)} \leq 0$$



$$-2 < t < -1 \quad \vee \quad 1 \leq t \leq 2$$

$$-2 < \log_2 x < -1 \quad \vee \quad 1 \leq \log_2 x \leq 2 \qquad \text{C.A. } x > 0$$

$$\boxed{\frac{1}{4} < x < \frac{1}{2} \quad \vee \quad 2 \leq x \leq 4}$$

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$$\frac{2}{\log_5^2 (x-4)} \leq 2 - \frac{3}{\log_5 (x-4)}$$

$$t = \log_5 (x-4)$$

$$\frac{2}{t^2} \leq 2 - \frac{3}{t}$$

$$\frac{2}{t^2} + \frac{3}{t} - 2 \leq 0$$

$$\frac{2+3t-2t^2}{t^2} \leq 0$$

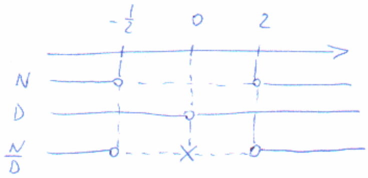
$$\frac{2t^2-3t-2}{t^2} \geq 0$$

$N > 0 \rightarrow 2t^2 - 3t - 2 > 0$

$$t_{1,2} = \frac{3 \pm 5}{4} = \left\langle \begin{array}{l} -\frac{1}{2} \\ 2 \end{array} \right.$$

$$t < -\frac{1}{2} \quad \vee \quad t > 2$$

$D > 0 \rightarrow t^2 > 0 \rightarrow t \neq 0$



$$t \leq -\frac{1}{2} \quad \vee \quad t \geq 2$$

$$\log_5 (x-4) \leq -\frac{1}{2} \quad \vee \quad \log_5 (x-4) \geq 2$$

$$\begin{cases} x > 4 \\ x - 4 \leq \frac{1}{\sqrt{5}} \end{cases} \quad \vee \quad \begin{cases} x > 4 \\ x - 4 \geq 25 \end{cases}$$

$$\boxed{4 < x \leq 4 + \frac{1}{\sqrt{5}} \quad \vee \quad x \geq 29}$$

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$$\frac{2}{\log_4^2(x+3)} \leq 2 - \frac{3}{\log_4(x+3)}$$

$$t = \log_4(x+3)$$

$$\frac{2}{t^2} - 2 + \frac{3}{t} \leq 0$$

$$\frac{2-2t^2+3t}{t^2} \leq 0$$

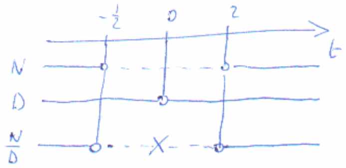
$$\frac{2t^2-3t-2}{t^2} \geq 0$$

$N>0 \qquad 2t^2-3t-2>0$

$$t_{1,2} = \frac{3\pm5}{4} = \begin{cases} -\frac{1}{2} \\ 2 \end{cases}$$

$$t<-\frac{1}{2} \quad \vee \quad t>2$$

$D>0 \qquad t^2>0 \rightarrow t\neq 0$



$$t \leq -\frac{1}{2} \quad \vee \quad t \geq 2$$

$$\log_4(x+3) \leq -\frac{1}{2} \quad \vee \quad \log_4(x+3) \geq 2$$

$$\begin{cases} x>-3 \\ x+3 \leq \frac{1}{2} \end{cases} \quad \vee \quad \begin{cases} x>-3 \\ x+3 \geq 16 \end{cases}$$

$$\boxed{-3<x\leq -\frac{5}{2} \quad \vee \quad x \geq 13}$$

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$$\sqrt{\log_2^2 x - 4} \geq \log_2 x + 1$$

$$t = \log_2 x$$

$$\sqrt{t^2-4} \geq t+1$$

$$\begin{cases} t^2-4 \geq 0 \\ t+1<0 \end{cases} \quad \vee \quad \begin{cases} t+1 \geq 0 \\ t^2-4 \geq t^2+2t+1 \end{cases}$$

$$\begin{cases} t\leq -2 \ \vee \ t \geq 2 \\ t<-1 \end{cases} \quad \vee \quad \begin{cases} t \geq -1 \\ t \leq -\frac{5}{2} \end{cases}$$

$$t \leq -2 \qquad\qquad \emptyset$$

$$t \leq -2$$

$$\log_2 x \leq \log_2 \frac{1}{4}$$

$$\begin{cases} x>0 \\ x \leq \frac{1}{4} \end{cases}$$

$$\boxed{0<x\leq \frac{1}{4}}$$

$$\sqrt{\log_3^2 x - 9} \geq \log_3 x + 1$$

$$t = \log_3 x$$

$$\sqrt{t^2-9} \geq t+1$$

$$\begin{cases} t^2-9 \geq 0 \\ t+1<0 \end{cases} \quad \vee \quad \begin{cases} t+1 \geq 0 \\ t^2-9 \geq t^2+2t+1 \end{cases}$$

$$\begin{cases} t\leq -3 \ \vee \ t \geq 3 \\ t<-1 \end{cases} \quad \vee \quad \begin{cases} t \geq -1 \\ t \leq -5 \end{cases}$$

$$t \leq -3 \qquad\qquad \emptyset$$

$$t \leq -3$$

$$\log_3 x \leq -3$$

$$\begin{cases} x>0 \\ x \leq \frac{1}{27} \end{cases}$$

$$\boxed{0<x\leq \frac{1}{27}}$$

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$$\sqrt{(\log_2 x-3)(\log_2 x-1)} \geq \log_2 x+2$$

$$t=\log_2 x$$

$$\sqrt{(t-3)(t-1)} \geq t+2$$

$$\begin{cases}(t-3)(t-1)\geq 0\\ t+2<0\end{cases} \vee \begin{cases}t+2\geq 0\\ \cancel{t^2}-4t+3\geq \cancel{t^2}+4t+4\end{cases}$$

$$\begin{cases}t\leq 1 \vee t\geq 3\\ t<-2\end{cases} \vee \begin{cases}t\geq -2\\ t\leq -\frac{1}{8}\end{cases}$$

$$t<-2 \quad \vee \quad -2\leq t\leq -\frac{1}{8}$$

$$t\leq -\frac{1}{8}$$

$$\log_2 x\leq -\frac{1}{8} \rightarrow \begin{cases}x>0\\ x\leq \frac{1}{\sqrt[8]{2}}\end{cases} \rightarrow \boxed{0<x\leq \frac{1}{\sqrt[8]{2}}}$$

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$$\sqrt{(\log_3 x-2)(\log_3 x-1)} \geq \log_3 x+3$$

$$t=\log_3 x$$

$$\sqrt{(t-2)(t-1)} \geq t+3$$

$$\begin{cases}(t-2)(t-1)\geq 0\\ t+3<0\end{cases} \vee \begin{cases}t+3\geq 0\\ \cancel{t^2}-3t+2\geq \cancel{t^2}+6t+9\end{cases}$$

$$\begin{cases}t\leq 1 \vee t\geq 2\\ t<-3\end{cases} \vee \begin{cases}t\geq -3\\ t\leq -\frac{7}{9}\end{cases}$$

$$t<-3 \quad \vee \quad -3\leq t\leq -\frac{7}{9}$$

$$t\leq -\frac{7}{9}$$

$$\log_3 x\leq -\frac{7}{9}$$

$$\log_3 x\leq \log_3 3^{-\frac{7}{9}}$$

$$\begin{cases}x>0\\ x\leq 3^{-\frac{7}{9}}\end{cases} \rightarrow \boxed{0<x\leq 3^{-\frac{7}{9}}}$$

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$$\log_3\left(3\cdot2^{2x}-2^x\right)-\log_3\left(2^x+1\right)\geq x\log_3 2 \qquad C.A.\quad 3\cdot2^{2x}-2^x>0$$

$$\log_3\left(3\cdot2^{2x}-2^x\right)\geq \log_3 2^x+\log_3\left(2^x+1\right) \qquad 2^x\left(3\cdot2^x-1\right)>0$$

$$\log_3\left(3\cdot2^{2x}-2^x\right)\geq \log_3\left(2^{2x}+2^x\right) \qquad 2^x>\frac{1}{3}$$

$$x>-\frac{\log 3}{\log 2}$$

$$3\cdot2^{2x}-2^x\geq 2^{2x}+2^x$$

$$2\cdot2^{2x}-2\cdot2^x\geq 0$$

$$2\cdot2^x\left(2^x-1\right)\geq 0 \;\rightarrow\; 2^x\geq 1 \;\rightarrow\; \boxed{x\geq 0}$$

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$$\log_2\left(2\cdot3^{2x}-3^x\right)-\log_2\left(3^x+1\right)\geq x\log_2 3 \qquad C.A.\quad 2\cdot3^{2x}-3^x>0$$

$$\log_2\left(2\cdot3^{2x}-3^x\right)\geq \log_2 3^x+\log_2\left(3^x+1\right) \qquad 3^x\left(2\cdot3^x-1\right)>0$$

$$\log_2\left(2\cdot3^{2x}-3^x\right)\geq \log_2\left(3^{2x}+3^x\right) \qquad 3^x>\frac{1}{2}$$

$$x>-\frac{\log 2}{\log 3}$$

$$2\cdot3^{2x}-3^x\geq 3^{2x}+3^x$$

$$3^{2x}-2\cdot3^x\geq 0$$

$$3^x\left(3^x-2\right)\geq 0 \;\rightarrow\; 3^x\geq 2 \;\rightarrow\; \boxed{x\geq \log_3 2}$$

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$$\log_2\left(6^{2x}-3\cdot6^x\right)\leq \log_2\left(6^x+3\right)+\log_2\left(6^x-4\right)$$

$$\begin{cases}6^{2x}-3\cdot6^x>0\\ 6^x-4>0\\ \log_2\left(6^{2x}-3\cdot6^x\right)\leq \log_2\left(6^{2x}-6^x-12\right)\end{cases} \rightarrow \begin{cases}6^x\left(6^x-3\right)>0\\ 6^x-4>0\\ 6^{2x}-3\cdot6^x\leq 6^{2x}-6^x-12\end{cases} \rightarrow$$

$$\rightarrow \begin{cases}x>\frac{\log 3}{\log 6}\\ x>\frac{\log 4}{\log 6}\\ 2\cdot6^x\geq 12\end{cases} \rightarrow \begin{cases}x>\frac{\log 3}{\log 6}\\ x>\frac{\log 4}{\log 6}\\ x\geq 1\end{cases} \rightarrow \boxed{x\geq 1}$$

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$$\log_2\left(4^{2x}-3\cdot4^x+6\right) \leq \log_2\left(4^x-2\right)+\log_2\left(4^x+1\right)$$

$$\begin{cases} 4^{2x}-3\cdot4^x+6>0 \\ 4^x-2>0 \\ \log_2\left(4^{2x}-3\cdot4^x+6\right) \leq \log_2\left(4^{2x}-4^x-2\right) \end{cases} \rightarrow \begin{cases} \forall x\in\mathbb{R} \\ x>\frac{\log 2}{\log 4} \\ \cancel{4^{2x}}-3\cdot4^x+6 \leq \cancel{4^{2x}}-4^x-2 \end{cases} \rightarrow$$

$$\rightarrow \begin{cases} x>\frac{\log 2}{\log 4} \\ 2\cdot4^x\geq 8 \end{cases} \rightarrow \begin{cases} x>\frac{\log 2}{\log 4} \\ 4^x\geq 4 \end{cases} \rightarrow \begin{cases} x>\frac{\log 2}{\log 4} \\ x\geq 1 \end{cases} \rightarrow \boxed{x\geq 1}$$

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$$\log_{\frac{1}{2}}\left(7-2^x\right)-\log_{\frac{1}{2}}\left(5+4^x\right)+\log_{\frac{1}{2}} 7 \geq 0$$

$$\log_{\frac{1}{2}}\left(49-7\cdot2^x\right) \geq \log_{\frac{1}{2}}\left(5+4^x\right)$$

$$49-7\cdot2^x \leq 5+2^{2x}$$

$$2^{2x}+7\cdot2^x-44\geq 0 \qquad 2^x=\frac{-7\pm15}{2}=\begin{cases}-11\\4\end{cases}$$

$$2^x\leq -11 \quad \vee \quad 2^x\geq 4$$

$$\begin{cases} 2^x\geq 4 \\ x<\frac{\log 7}{\log 2} \quad (C.A.) \end{cases} \rightarrow \begin{cases} x\geq 2 \\ x<\frac{\log 7}{\log 2}\approx 2,807 \end{cases} \rightarrow 2\leq x<\frac{\log 7}{\log 2}$$

C.A.

$$7-2^x>0$$

$$2^x<7$$

$$x<\frac{\log 7}{\log 2}$$

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$$\log_{2-x}(x-1)>1$$

$$\frac{\log(x-1)}{\log(2-x)}-1>0$$

$$\frac{\log(x-1)-\log(2-x)}{\log(2-x)}>0$$

$N>0$

$$\log(x-1)>\log(2-x)$$

$$\begin{cases} x-1>0 \\ 2-x>0 \\ x-1>2-x \end{cases} \rightarrow \begin{cases} x>1 \\ x<2 \\ x>\frac{3}{2} \end{cases}$$

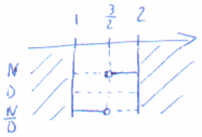
$$\frac{3}{2}<x<2$$

$D>0$

$$\log(2-x)>0$$

$$\begin{cases} 2-x>0 \\ 2-x>1 \end{cases} \rightarrow \begin{cases} x<2 \\ x<1 \end{cases}$$

$$x<1$$



$$\boxed{1<x<\frac{3}{2}}$$

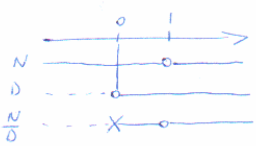
$$\log_2 x+\log_x 2<2$$

$$\log_2 x+\frac{1}{\log_2 x}-2<0 \qquad t=\log_2 x$$

$$t+\frac{1}{t}-2<0$$

$$\frac{t^2-2t+1}{t}<0$$

$$\frac{(t-1)^2}{t}<0$$



$$t<0$$

$$\log_2 x<0$$

$$\log_2 x<\log_2 1$$

$$\begin{cases} x>0 \\ x<1 \end{cases} \rightarrow \boxed{0<x<1}$$

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$$81\log^4_{\frac{1}{3}} x + \log^2_{\frac{1}{2}} x - 2 \geq 0$$

$$81\cdot\left(\frac{\log_{\frac{1}{2}} x}{\log_{\frac{1}{2}}\frac{1}{8}}\right)^4 + \log^2_{\frac{1}{2}} x - 2 \geq 0$$

$$81\cdot\left(\frac{\log_{\frac{1}{2}} x}{3}\right)^4 + \log^2_{\frac{1}{2}} x - 2 \geq 0$$

$$\log^4_{\frac{1}{2}} x + \log^2_{\frac{1}{2}} x - 2 \geq 0$$

$$z = \log^2_{\frac{1}{2}} x$$

$$z^2 + z - 2 \geq 0$$

$$z_{1,2} = \frac{-1 \pm 3}{2} = \begin{cases} -2 \\ 1 \end{cases}$$

$$z \leq -2 \quad \vee \quad z \geq 1$$

$$\log^2_{\frac{1}{2}} x \leq -2 \quad \vee \quad \log^2_{\frac{1}{2}} x \geq 1$$

impossibile $\quad \log_{\frac{1}{2}} x \leq -1 \quad \vee \quad \log_{\frac{1}{2}} x \geq 1$

$$\begin{cases} x > 0 \\ x \geq 2 \end{cases} \quad \vee \quad \begin{cases} x > 0 \\ x \leq \frac{1}{2} \end{cases}$$

$$\boxed{0 < x \leq \frac{1}{2} \quad \vee \quad x \geq 2}$$

$$\log_{\frac{27}{8}}\left(1 - \frac{3}{x}\right) < -\frac{1}{3}$$

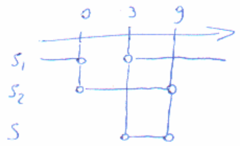
$$\begin{cases} 1 - \frac{3}{x} > 0 \\ \log_{\frac{27}{8}}\left(1 - \frac{3}{x}\right) < \log_{\frac{27}{8}}\left(\frac{27}{8}\right)^{-\frac{1}{3}} \end{cases}$$

$$\begin{cases} \frac{x-3}{x} > 0 \\ 1 - \frac{3}{x} < \frac{2}{3} \end{cases}$$

$$\begin{cases} x < 0 \quad \vee \quad x > 3 \\ \frac{1}{3} - \frac{3}{x} < 0 \end{cases}$$

$$\begin{cases} x < 0 \quad \vee \quad x > 3 \\ \frac{x-9}{3x} < 0 \end{cases}$$

$$\begin{cases} x < 0 \quad \vee \quad x > 3 \\ 0 < x < 9 \end{cases}$$



$$\boxed{3 < x < 9}$$

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$$\frac{2 + \log_2 x}{2\log_2 x - 1} - 3 + \frac{1 + 3\log_2 x}{2 + \log_2 x} > 0$$

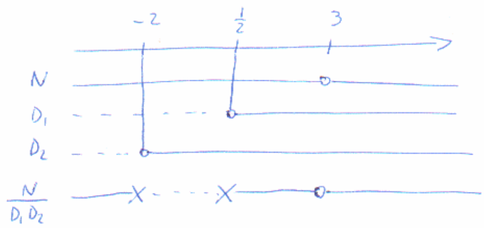
$$t = \log_2 x$$

$$\frac{2+t}{2t-1} - 3 + \frac{1+3t}{2+t} > 0$$

$$\frac{t^2 + 4t + 4 \cancel{-6t^2} - 9t + 6 \cancel{+6t^2} - t - 1}{(2t-1)(2+t)} > 0$$

$$\frac{t^2 - 6t + 9}{(2t-1)(2+t)} > 0$$

$$\frac{(t-3)^2}{(2t-1)(2+t)} > 0$$



$$t < -2 \quad \vee \quad \frac{1}{2} < t < 3 \quad \vee \quad t > 3$$

$$\log_2 x < \log_2 \frac{1}{4} \quad \vee \quad \log_2 2^{\frac{1}{2}} < \log_2 x < \log_2 8 \quad \vee \quad \log_2 x > \log_2 8$$

$$\begin{cases} x > 0 \\ x < \frac{1}{4} \end{cases} \quad \vee \quad \begin{cases} x > 0 \\ \sqrt{2} < x < 8 \end{cases} \quad \vee \quad \begin{cases} x > 0 \\ x > 8 \end{cases}$$

$$\boxed{0 < x < \frac{1}{4} \quad \vee \quad \sqrt{2} < x < 8 \quad \vee \quad x > 8}$$

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$$4^x - 6 \cdot 10^x + 9 \cdot 25^x > 0$$

$$(2^x)^2 - 6 \cdot 2^x \cdot 5^x + 9 (5^x)^2 > 0$$

$$(2^x - 3 \cdot 5^x)^2 > 0$$

$$2^x - 3 \cdot 5^x \neq 0$$

$$\left(\frac{2}{5}\right)^x \neq 3 \rightarrow x \neq \log_{\frac{2}{5}} 3$$

$$\log_{\frac{1}{2}} \frac{|x|+1}{2+|x|} > 1 \qquad \frac{|x|+1}{2+|x|} > 0 \quad \forall x \in \mathbb{R}$$

$$\log_{\frac{1}{2}} \frac{|x|+1}{2+|x|} > \log_{\frac{1}{2}} \frac{1}{2}$$

$$\frac{|x|+1}{2+|x|} < \frac{1}{2}$$

$$2|x|+2 < 2+|x|$$

$$|x| < 0 \qquad \boxed{S = \emptyset}$$

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$$\log_{\frac{1}{2}} \frac{3|x|+1}{2+|x|} < 1$$

$$\frac{3|x|+1}{2+|x|} > \frac{1}{2}$$

$$6|x|+2 > 2+|x|$$

$$5|x| > 0 \rightarrow \boxed{\forall x \neq 0}$$

$$\log_{\frac{1}{2}} \frac{3+|x+3|}{|x-1|-2} > 0$$

$$\begin{cases} |x-1|-2 > 0 \\ \log_{\frac{1}{2}} \frac{3+|x+3|}{|x-1|-2} > \log_{\frac{1}{2}} 1 \end{cases}$$

$$\begin{cases} x < -1 \quad \vee \quad x > 3 \\ \frac{3+|x+3|}{|x-1|-2} < 1 \end{cases}$$

2ª diseq. del sistema:

$$\frac{3+|x+3|}{|x-1|-2} < 1$$

per C.A. il denominatore $|x-1|-2$ è maggiore di zero

$$3+|x+3| < |x-1|-2$$

$$5+|x+3| < |x-1|$$

$$\begin{cases} x < -3 \\ 5-x-3 < -x+1 \end{cases} \vee \begin{cases} -3 \leq x < 1 \\ 5+x+3 < -x+1 \end{cases} \vee \begin{cases} x \geq 1 \\ 5+x+3 < x-1 \end{cases}$$

$$\begin{cases} x < -3 \\ \emptyset \end{cases} \vee \begin{cases} -3 \leq x < 1 \\ 2x < -7 \end{cases} \vee \begin{cases} x \geq 1 \\ \emptyset \end{cases}$$

$$\emptyset \qquad \begin{cases} -3 \leq x < 1 \\ x < -\frac{7}{2} \end{cases} \qquad \emptyset$$

$$\emptyset$$

$$\boxed{S = \emptyset}$$

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$$|\log_a x - 2| - \log_a^2 x > 0, \qquad a \in \mathbb{R}^+ - \{1\}$$

$$t = \log_a x$$

$$|t-2| - t^2 > 0$$

$$\begin{cases} t-2 \geq 0 \\ t-2-t^2 > 0 \end{cases} \vee \begin{cases} t-2 < 0 \\ -t+2-t^2 > 0 \end{cases}$$

$$\begin{cases} t \geq 2 \\ t^2-t+2 < 0 \end{cases} \vee \begin{cases} t < 2 \\ t^2+t-2 < 0 \end{cases}$$

$$\begin{cases} t \geq 2 \\ \phi \end{cases} \vee \begin{cases} t < 2 \\ -2 < t < 1 \end{cases}$$

$$-2 < t < 1$$

$a > 1$: $-2 < t < 1$

$$-2 < \log_a x < 1$$

$$\log_a a^{-2} < \log_a x < \log_a a$$

$$\begin{cases} x > 0 \\ \frac{1}{a^2} < x < a \end{cases}$$

$$\boxed{a^{-2} < x < a}$$

$0 < a < 1$: $-2 < t < 1$

$$-2 < \log_a x < 1$$

$$\log_a a^{-2} < \log_a x < \log_a a$$

$$\begin{cases} x > 0 \\ a < x < a^{-2} \end{cases}$$

$$\boxed{a < x < a^{-2}}$$

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$$\left|\left|\log_{\frac{1}{2}}(x+2)-3\right|-2\right|<1 \qquad t=\log_{\frac{1}{2}}(x+2)$$

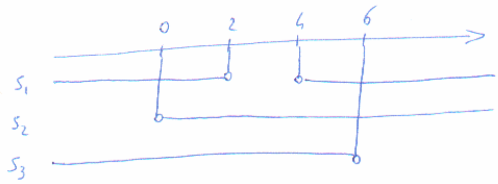
$$\left||t-3|-2\right|<1$$

$$\begin{cases}|t-3|-2>-1\\|t-3|-2<1\end{cases}$$

$$\begin{cases}|t-3|>1\\|t-3|<3\end{cases}$$

$$\begin{cases}t-3<-1 \quad \vee \quad t-3>1\\t-3>-3\\t-3<3\end{cases}$$

$$\begin{cases}t<2 \quad \vee \quad t>4\\t>0\\t<6\end{cases}$$



$$0<t<2 \quad \vee \quad 4<t<6$$

$$0<\log_{\frac{1}{2}}(x+2)<2 \quad \vee \quad 4<\log_{\frac{1}{2}}(x+2)<6$$

$$\begin{cases}x+2>0\\\log_{\frac{1}{2}}1<\log_{\frac{1}{2}}(x+2)<\log_{\frac{1}{2}}\frac{1}{4}\end{cases} \vee \begin{cases}x+2>0\\\log_{\frac{1}{2}}\frac{1}{16}<\log_{\frac{1}{2}}(x+2)<\log_{\frac{1}{2}}\frac{1}{64}\end{cases}$$

$$\begin{cases}x>-2\\\frac{1}{4}<x+2<1\end{cases} \vee \begin{cases}x>-2\\\frac{1}{64}<x+2<\frac{1}{16}\end{cases}$$

$$\begin{cases}x>-2\\-\frac{7}{4}<x<-1\end{cases} \vee \begin{cases}x>-2\\-\frac{127}{64}<x<-\frac{31}{16}\end{cases}$$

$$\boxed{-\frac{127}{64}<x<-\frac{31}{16} \quad \vee \quad -\frac{7}{4}<x<-1}$$